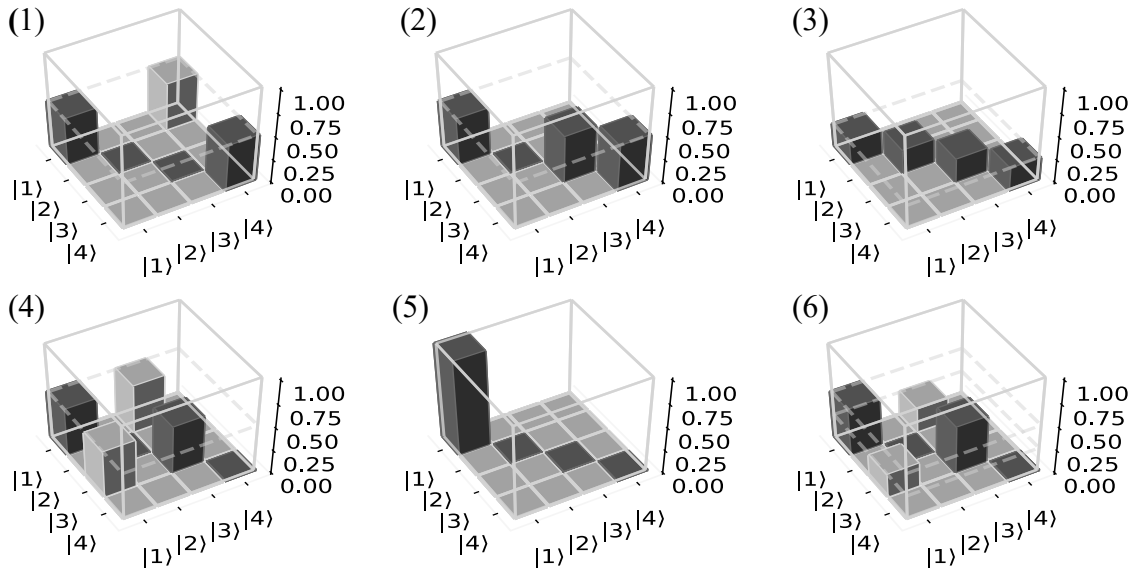


Problem I (10%)



The above figure shows a number of "density matrices" for a 4-level system. The dark grey elements along the diagonal are populations; the light gray off-diagonal elements are coherences. The height of the columns indicate the absolute values of the relevant populations and coherences.

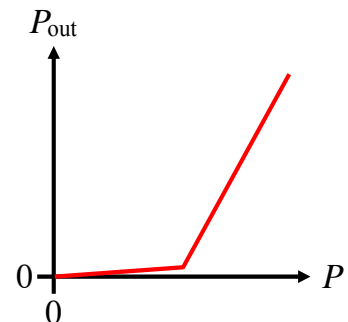
- (a) Some of the density matrices correspond to pure states, some to mixed states, and some are unphysical. Indicate which are which. No explanation needed. (10%)

Hint: These density matrices can be sorted into categories by little more than visual inspection.

Problem II (30%)

The figure on the left shows the output power P_{out} of a laser versus the pumping rate P .

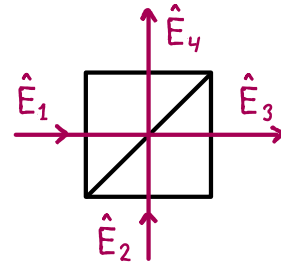
- (a) Explain (in words) the major features of the plot. (10%)
- (b) What does the plot tell us about the laser pumping scheme (3 vs 4 level)? Explain (in words). (10%)
- (c) Assuming there are no other losses in the cavity, redraw the plot to show what would happen if we doubled the reduced the transmittivity T of the output mirror by a factor of 2. (10%)



Hint: Focus on the general behavior above and below the kink and don't fret about its precise shape. No math required.

Problem III (25%)

- (a) Consider a 4-port beam splitter as depicted, with transmission and reflection coefficients t and r . The input state is $|\Psi_{\text{in}}\rangle = |1\rangle_1 |0\rangle_2$ (one photon in port 1 and zero photons in port 2). Find the output state $|\Psi_{\text{out}}\rangle$ in terms of t , r , and number states in ports 3 and 4. (10%)



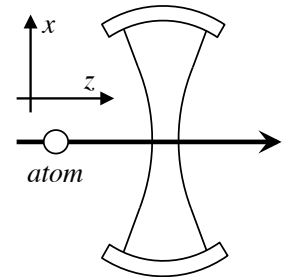
This simple example tells us that photon number states are very sensitive to loss. Even in the simplest possible case, after passing through a lossy medium (here modeled by a beam splitter) we no longer have a photon number state, but rather a statistical mixture of different number states. And the generalization to large number states, $|\Psi_{\text{in}}\rangle = |n\rangle_1 |0\rangle_2$, results in hugely complicated output states with massive photon-number - mode entanglement.

- (c) Show that a coherent state is robust in the presence of loss. That is, after passing through a lossy medium it remains a coherent state, though with reduced amplitude. (15%)

Problem IV (35%)

Consider an optical cavity as shown in the figure. A 2-level atom, on-resonance with the cavity mode and moving along the z axis with velocity v , crosses the cavity with a transit time much shorter than the time scales for atom decay and cavity photon loss. The atom-light coupling is characterized by a Rabi frequency $g(z) = 2g\sqrt{n} \exp(-z^2/2\sigma^2)$, where g is the vacuum Rabi frequency and n is the number of excitations in the system.

- (a) First describe conceptually (in words) how you might use a stream of n two-level atoms as a way to deposit exactly n photons in the initially empty cavity. (25%)
- (b) Find the velocity with which an atom initially in the excited state must cross the empty cavity to leave its excitation behind in the form of a photon. (10%)



Hint: Think about the time dependent atom-cavity interaction along the path $g(z = tv)$ and how it fits with the pulse area theorem.

Useful math: $\int_{-\infty}^{\infty} \exp(-a^2/2b^2) da = \sqrt{2\pi}b$