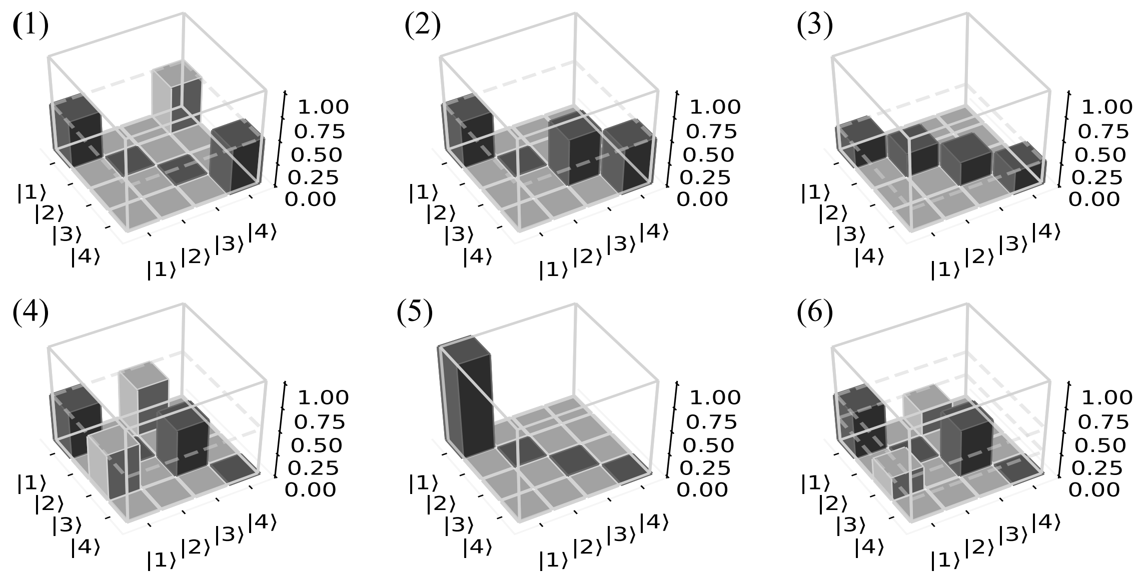
3OPTI 544 Final Exam, May 6, 2022 Jessen

Problem I (10%)



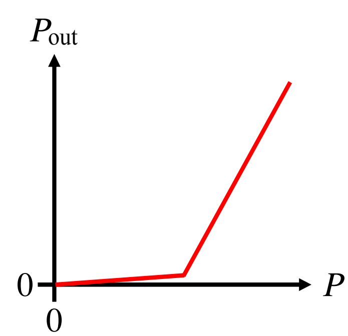
The above figure shows a number of "density matrices" for a 4-level system. The dark grey elements along the diagonal are populations; the light gray off-diagonal elements are coherences. The height of the columns indicate the absolute values of the relevant populations and coherences.

(a) Some of the density matrices correspond to pure states, some to mixed states, and some are unphysical. Indicate which are which. No explanation needed. (10%)

Hint: These density matrices can be sorted into categories by little more that visual inspection.

Problem II (30%)

The figure on the left shows the output power P_{out} of a laser versus the pumping rate P.



(a) Explain (in words) the major features of the plot. (10%)

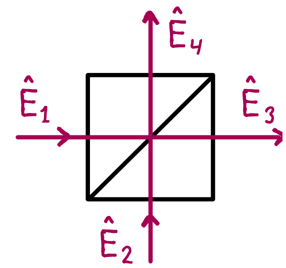
(b) What does the plot tell us about the laser pumping scheme (3 vs 4 level)? Explain (in words). (10%)

(c) Assuming there are no other losses in the cavity, redraw the plot to show what would happen if we reduced the transmittivity T of the output mirror by a factor of 2. (10%)

Hint: Focus on the general behavior above and below the kink and don't fret about its precise shape. No math required.

Problem III (25%)

(a) Consider a 4-port beam splitter as depicted, with transmission and reflection coefficients t and r. The input state is $|\Psi_{\text{in}}\rangle = |1\rangle_1 |0\rangle_2$ (one photons in port 1 and zero photons in port 2). Find the output state $|\Psi_{\text{out}}\rangle$ in terms of t, r, and number states in ports 3 and 4. (10%)



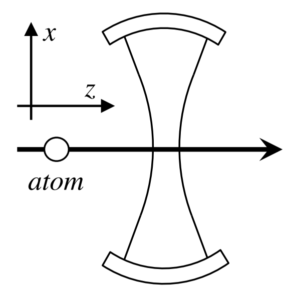
This simple example tells us that photon number states are very sensitive to loss. Even in the simplest possible case, after passing through a lossy medium (here modeled by a beam splitter) we no longer have a photon number state, but rather a statistical mixture of different number states. And the generalization to large number states, $|\Psi_{\text{in}}\rangle = |n\rangle_1 |0\rangle_2$, results in hugely complicates output states with massive photon-number - mode entanglement.

(b) Show that a coherent state is robust in the presence of loss. That is, after passing through a lossy medium it remains a coherent state, though with reduced amplitude. (15%)

Problem IV (35%)

Consider an optical cavity as shown in the figure. A 2-level atom, on-resonance with the cavity mode and moving along the z axis with velocity v, crosses the cavity with a transit time much shorter than the time scales for atom decay and cavity photon loss. The atom-light coupling is characterized by a vacuum Rabi frequency $2g(z) = 2g\,\exp(-z^2/2\sigma^2)$, where g is the vacuum Rabi frequency and n is the number of excitations in the system.

(a) First describe conceptually (in words) how you might use a stream of n two-level atoms as a way to deposit exactly n photons in the initially empty cavity. (25%)



(b) Find the velocity with which an atom initially in the excited state must cross the empty cavity to leave its excitation behind in the form of a photon. (10%)

Hint: Think about the time dependent atom-cavity interaction along the path $g(z = t\,v)$ and how it fits with the pulse area theorem.

Useful math: $\int_{-\infty}^{\infty} \exp(-a^2/2b^2)\,da = \sqrt{2\pi b^2}$