

OPTI 544: Homework Set #9
Posted April 25, Due May 2.

- Keep a copy of your Solution Set -

I

A Cs atom with transition wavelength $\lambda = 852\text{nm}$ and excited state decay rate $A_{21} = 3.28 \times 10^7/\text{s}$ is trapped at an antinode of the field in a resonant optical cavity with length $l = 100\mu\text{m}$ and effective cross sectional area $A = \pi (2\mu\text{m})^2$. We assume the field polarization is parallel to the electric dipole matrix element, and that the cavity mirrors are perfect, lossless reflectors.

- (a) Calculate the vacuum Rabi frequency g and compare to A_{21} . Is the system in a regime that permits the observation of coherent vacuum Rabi oscillations?
- (b) A weak probe beam is passed through the cavity. Sketch the transmission as the probe frequency ω is tuned around the atom/cavity resonance frequency ω_0 . What phenomenon are you seeing?

II

In the following, consider a normal mode of the electromagnetic field in a cavity that contains *two* identical 2-level atoms. For simplicity we assume the cavity mode is resonant with the atomic transition frequency, $\omega = \omega_{21}$, and that the interaction strength g is the same for both atoms.

- (a) Write down the single-mode, two-atom Jaynes-Cummings Hamiltonian in terms of creation and annihilation operators for the field, and atomic operators $\hat{\sigma}_{\pm,c}^{(i)}$, where the index $i = 1, 2$ refers to the two atoms.
- (b) The Hamiltonian from (a) has the form $\hat{H} = \hat{H}_0 + \hat{H}_{AF}$. Use the notation $|j, k, n\rangle$ for the eigenstates of $\hat{H}_0$, where j, k are the states of the first and second atom and n is the photon number, respectively. Find the eigenvalues λ and $\lambda_q, q = 0, \pm 1$ of $\hat{H}$ corresponding to coupled atom-cavity states that have zero or one quantum of excitation, respectively. Then draw a level diagram showing the dressed-state energies for the zero- and one-excitation parts of the two-atom Jaynes-Cummings ladder.
- (c) Sketch the expected transmission of a weak probe beam through the coupled atom-cavity system as the probe frequency is tuned from well below to well above the cavity mode frequency ω . No formal calculation is required; rely instead on your physical intuition and your results in (b).
- (d) Returning to (c), find the eigenstate associated with the eigenvalue λ_0 in the single-excitation manifold. Based on your result, how might you revise your sketch from (d)?