OPTI 544 1st Midterm Exam, March 4, 2022

Problem 1

(a) During lectures we looked at the level structure for the 1S to 2P transition in Hydrogen, including fine and hyperfine structure. We eventually picked the $1S_{1/2}$ (F=1) to $2P_{3/2}$ (F=2) transition. Draw a similar level diagram, but for the $1S_{1/2}$ (F=1) to $2P_{1/2}$ (F=1) transition. (10%)

(b) Assume the atom is interacting with $\vec{\varepsilon}_x$ polarized light (linear polarization along the x-axis). Indicate on your level diagram those transitions between the magnetic sublevels that are electric dipole allowed for this polarization. (15%)

Problem 2

(a) Write out (do not derive) the expressions for the imaginary index of refraction, $n_1(\omega_0)$, and the absorption coefficient $a(\omega_0)$, in the form appropriate for near-resonance excitation of a weakly polarizable medium of Lorentz atoms with transition frequency ω_0. (15%)

(b) Write out (do not derive) the expression for the absorption coefficient $a(\omega_{21})$ in a medium of two-level atoms with transition frequency ω_{21}, assuming there are no collisions and the light intensity is well below I_{sat}. (10%)

Problem 3

A subset of the postulates of quantum mechanics say:

(a) A unique quantum state is described by a unique state vector $|\psi\rangle$, where $\langle\psi|\psi\rangle = 1$.

(b) Let A be an observable with eigenvalues a and corresponding eigenvetors $|a\rangle$. Given the state $|\psi\rangle$, the expectation value of this observable is $\langle A\rangle = \langle\psi|A|\psi\rangle$.

(c) The probability of getting the outcome a when measuring A is $\mathcal{P}(a) = |\langle\psi|a\rangle|^2$. (Here and in the following we assume for simplicity that the eigenvalue a is non-degenerate.)

(d) When a measurement of the observable A has the outcome a, the state vector immediately afterwards is the eigenstate $|a\rangle$.

(e) The state vector $|\psi\rangle$ evolves according to the Schrödinger equation, $i\hbar\frac{d}{dt}|\psi\rangle = H|\psi\rangle$, where H is the Hamiltonian.

Write out equivalent expressions (a)-(e) when using the density matrix formalism. You may rely on results from class and/or the notes. (10% each)