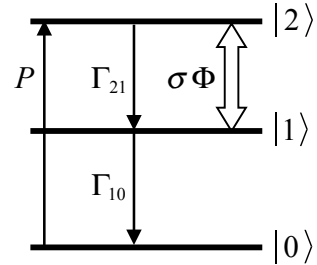# OPTI 544 2nd Midterm Exam, April 14, 2021

I

Consider a laser with a 3-level pumping scheme as indicated by the figure on the right, with lasing occurring on the transition between levels $|1\rangle$ and $|2\rangle$. To simplify things a bit we assume the pumping mechanism is irreversible, i. e., it transfers atoms from $|0\rangle$ to $|2\rangle$ but not the other way.

(a) Write down a set of rate equations for the number densities N_0, N_1, and N_2 in terms of the pumping rate, the spontaneous decay rates, and the rates of absorption and stimulated emission. (15%)

(b) What is the steady state value of the intra-cavity photon flux Φ below threshold? (5%)

(c) Use your answers in (a) and (b) to determine the conditions on the various rates required to achieve a steady state population inversion on the lasing transition. (15%)

(d) Write down expressions for the small-signal gain, the saturated gain, and the saturation flux. (15%)

II

A quantum beamsplitter has inputs $\hat{E}_H(\tau<0)$, $\hat{E}_V(\tau<0)$, outputs $\hat{E}_H(\tau>0)$, $\hat{E}_V(\tau>0)$, transmission $t=\sqrt{3}/2$, and reflection $r=i/2$. In the Heisenberg picture the arguments $\tau<0$ and $\tau>0$ refer to times well before and after the light has entered and exited the region around the beamsplitter. This emphasizes the beamsplitter as a device that couples one "horizontal" mode to one "vertical" mode, and maps the input to output accordingly.
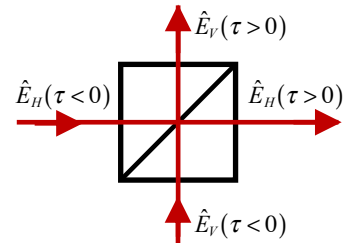
(a) Write out expressions for the output fields in terms of the input fields. Then write out the expressions for the input creation operators in terms of the output creation operators. (15%)

(b) Assume the input state is $|\Psi(\tau<0)\rangle=|0\rangle_H|1\rangle_V$. Find and write out an expression for the output state $|\Psi(\tau>0)\rangle$. (10%)

Looking at the possible inputs and outputs, it is evident that we are dealing with a two-level system, where states $|1\rangle$ and $|2\rangle$ correspond to having the photon in the horizontal and vertical modes, respectively.

(c) Find and write down an expression for the Bloch vectors corresponding to the input and output states. Then make a sketch that shows clearly how they are oriented relative to the Bloch sphere. Follow standard convention ($|1\rangle$ at the south pole and $|2\rangle$ at the north pole) and be sure to indicate the $\vec{i},\vec{j},\vec{k}$ vectors. (25%)