Quantized Light – Matter Interactions

With this notation

$$\hat{H}_{AF} = \hbar \sum_{\vec{k}} \left(g_{\vec{k}} \hat{\sigma}_+ \hat{a}_{\vec{k}} + \cancel{g_{\vec{k}} \hat{\sigma}_+ \hat{a}^+_{\vec{k}}} + \cancel{g^*_{\vec{k}} \hat{\sigma}_- \hat{a}_{\vec{k}}} + g^*_{\vec{k}} \hat{\sigma}_- \hat{a}^+_{\vec{k}} \right) \quad (4)$$

Energy conservation?

$$\hat{H}_{AF} = \hbar \sum_{\vec{k}} \left(g_{\vec{k}} \hat{\sigma}_+ \hat{a}_{\vec{k}} + g^*_{\vec{k}} \hat{\sigma}_- \hat{a}^+_{\vec{k}} \right)$$

Putting it all together

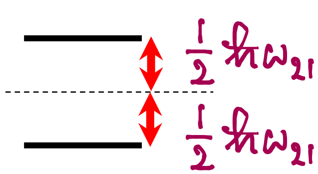
$$\boxed{\begin{aligned} \hat{H} &= \hat{H}_F + \hat{H}_A + \hat{H}_{AF} = \\ &\sum_{\vec{k}} \hbar\omega_{\vec{k}} \hat{a}^+_{\vec{k}} \hat{a}^+_{\vec{k}} + \frac{1}{2}\hbar\omega_{21}\hat{\sigma}_z + \sum_{\vec{k}} \hbar \left(g_{\vec{k}} \hat{\sigma}_+ \hat{a}_{\vec{k}} + g^*_{\vec{k}} \hat{\sigma}_- \hat{a}^+_{\vec{k}} \right) \end{aligned}} \quad (5)$$

Foundational result for remainder of course

We changed the zero point for energy by subtracting

$\sum_{\vec{k}} \frac{1}{2}\hbar\omega_{\vec{k}}$ and $\frac{1}{2}(E_2 - E_1)$

field — atom



Quantized Light – Matter Interactions

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$$\hat{H}_{AF} = \hbar \sum_{\vec{k}} \left(g_{\vec{k}} \hat{\sigma}_+ \hat{a}_{\vec{k}} + \cancel{g_{\vec{k}} \hat{\sigma}_+ \hat{a}^+_{\vec{k}}} + \cancel{g^*_{\vec{k}} \hat{\sigma}_- \hat{a}_{\vec{k}}} + g^*_{\vec{k}} \hat{\sigma}_- \hat{a}^+_{\vec{k}} \right)$$

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Putting it all together

$$\boxed{\hat{H} = \hat{H}_F + \hat{H}_A + \hat{H}_{AF} = \sum_{\vec{k}} \hbar \omega_{\vec{k}} \hat{a}^+_{\vec{k}} \hat{a}_{\vec{k}} + \frac{1}{2} \hbar \omega_{21} \hat{\sigma}_z + \sum_{\vec{k}} \hbar \left(g_{\vec{k}} \hat{\sigma}_+ \hat{a}_{\vec{k}} + g^*_{\vec{k}} \hat{\sigma}_- \hat{a}^+_{\vec{k}} \right)} \quad (5)$$

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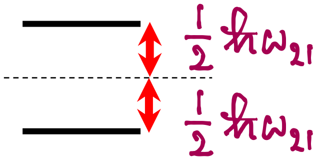
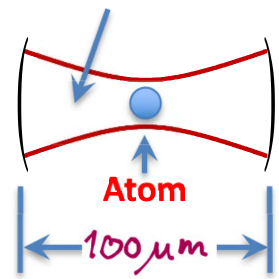


Interaction with Single-mode Fields

Good approx. in small, high-Q Cavity



$c/2L \gg A_{21}$

$|g_{\vec{k}}| \gg A_{21}, \gamma$

Single-mode (Jaynes-Cummings) Hamiltonian

$$\boxed{\hat{H} = \underbrace{\hbar \omega \hat{a}^+ \hat{a} + \frac{1}{2} \hbar \omega_{21} \hat{\sigma}_z}_{H_0} + \underbrace{\hbar \left(g_{\vec{k}} \hat{\sigma}_+ + g^*_{\vec{k}} \hat{\sigma}_- \right) \left(\hat{a}_{\vec{k}} + \hat{a}^+_{\vec{k}} \right)}_{H_{AF}}}$$

Quantized Light – Matter Interactions

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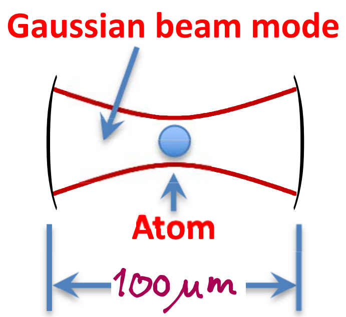

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Paradigm for spin-1/2 coupled to QHO

- Atom in high-Q cavity *)
- Quantum dot in high-Q
- Rydberg atom in superconducting μw Cavity
- Superconducting qubit in superconducting μw cavity
- Superconducting qubit in superconducting μw stripline cavity (circuit QED)
- Trapped ion with quantized COM motion *)

*) Nobel Prize in Physics 2012

Quantized Light – Matter Interactions

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More about Single-mode Cavity QED

$$\hat{H} = \underbrace{\hbar\omega\hat{a}^{+}\hat{a} + \frac{1}{2}\hbar\omega_{21}\hat{\sigma}_{z}}_{H_0} + \underbrace{\hbar\left(g_{\vec{k}}\hat{\sigma}_{+} + g^{*}_{\vec{k}}\hat{\sigma}_{-}\right)\left(\hat{a}_{\vec{k}} + \hat{a}^{+}_{\vec{k}}\right)}_{H_{AF}}$$

For simplicity $\vec{\eta}_{21} = \vec{\eta}_{12} \Rightarrow g_{\vec{k}} = g^{*}_{\vec{k}} = g_{\vec{k}}$

Note: $\hat{H}_{AF}$ conserves excitation number, couples $|2,n\rangle \leftrightarrow |1,n+1\rangle$

Series of 2-level systems, one for each n

All 2-level systems are alike
Rabi problem!

Switch to Interaction Picture, 6-7 in Notes:

$$\hat{H}_S \rightarrow \hat{H}_I = e^{i\frac{\hat{H}_0}{\hbar}t}\hat{H}_{AF}e^{-i\frac{\hat{H}_0}{\hbar}t}$$

$$|\psi_S(t)\rangle \rightarrow |\psi_I(t)\rangle = e^{i\frac{\hat{H}_0}{\hbar}t}|\psi_S(t)\rangle$$

Quantized Light – Matter Interactions

More about Single-mode Cavity QED

$$\hat{H} = \underbrace{\hbar\omega\,\hat{a}^+\hat{a} + \tfrac{1}{2}\hbar\omega_{21}\hat{\sigma}_z}_{H_0} + \underbrace{\hbar(g_{\vec{k}}\hat{\sigma}_+ + g_{\vec{k}}^*\hat{\sigma}_-)(\hat{a}_{\vec{k}} + \hat{a}_{\vec{k}}^+)}_{H_{AF}}$$

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Can show

$$e^{i\omega\hat{a}^+\hat{a}t}\,\hat{a}\,e^{-i\omega\hat{a}^+\hat{a}t} = \hat{a}\,e^{-i\omega t}$$

$$e^{i\frac{\omega_{21}}{2}\hat{\sigma}_z t}\,\hat{\sigma}_+\,e^{-i\frac{\omega_{21}}{2}\hat{\sigma}_z t} = \hat{\sigma}_+\,e^{i\omega_{21}t}$$

$$\hat{H}_{AF} = \hbar g\left(\hat{\sigma}_+\hat{a}\,e^{i(\omega_{21}-\omega)t} + \hat{\sigma}_+\hat{a}^+e^{i(\omega_{21}+\omega)t} + \hat{\sigma}_-\hat{a}\,e^{-i(\omega_{21}+\omega)t} + \hat{\sigma}_-\hat{a}^+e^{-i(\omega_{21}-\omega)t}\right)$$

RWA and resonant approximation

⇓

Jaynes-Cummings Hamiltonian

$$\boxed{\hat{H}_I = \hat{H}_{AF} = \hbar g\left(\hat{\sigma}_+\hat{a}\,e^{i\Delta t} + \hat{\sigma}_-\hat{a}^+e^{-i\Delta t}\right)}$$

$\Delta = \omega_{21} - \omega$

Quantized Light – Matter Interactions

More about Single-mode Cavity QED

$$\hat{H} = \underbrace{\hbar\omega\,\hat{a}^+\hat{a} + \tfrac{1}{2}\hbar\omega_{21}\hat{\sigma}_z}_{H_0} + \underbrace{\hbar(g_{\vec{k}}\hat{\sigma}_+ + g^*_{\vec{k}}\hat{\sigma}_-)(\hat{a}_{\vec{k}} + \hat{a}^+_{\vec{k}})}_{H_{AF}}$$

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Begin 04-26-2021

Eigenstates of $\hat{H}_0 = \hat{H}_F + \hat{H}_A$

State	Energy
$\|2,n\rangle$	$\hbar\omega n + \hbar\omega_{21}$
$\|1,n+1\rangle$	$\hbar\omega(n+1) - \hbar\omega_{21}$

Quantized Light – Matter Interactions

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Cavity QED version of the Rabi Problem

$$|\psi(0)\rangle = |2,n\rangle$$

$$|\psi(t)\rangle = C_{1,n+1}|1,n+1\rangle + C_{2,n}|2,n\rangle$$

Matrix elements

$$\langle 2,n|\hat{H}_{AF}|1,n+1\rangle = \hbar g\sqrt{n+1}\,e^{i\Delta t}$$

$$\langle 1,n+1|\hat{H}_{AF}|2,n\rangle = \hbar g\sqrt{n+1}\,e^{-i\Delta t}$$

Schrödinger Equation

$$i\hbar\frac{d}{dt}\begin{pmatrix} C_{1,n+1} \\ C_{2,n}\end{pmatrix} = \hbar g\sqrt{n+1}\begin{pmatrix} 0 & e^{-i\Delta t} \\ e^{i\Delta t} & 0\end{pmatrix}\begin{pmatrix} C_{1,n+1} \\ C_{2,n}\end{pmatrix}$$

Quantized Light – Matter Interactions

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$$\boxed{\begin{aligned} \dot{C}_{1,n+1} &= -ig\sqrt{n+1}\,e^{-i\Delta t}C_{2,n} \\ \dot{C}_{2,n} &= -ig\sqrt{n+1}\,e^{i\Delta t}C_{1,n+1} \end{aligned}}$$

Substitute $C_{1,n+1} \to C_1\,,\quad C_{2,n} \to C_2 e^{i\Delta t}$

Looks exactly like Semiclassical Rabi problem

Solve for $C_1(0) = 0\,,\quad C_2(0) = 1$

$$\boxed{\begin{aligned} C_{2,n}(t) &= \left[\cos\left(\frac{\Omega_n t}{2}\right) - i\frac{\Delta}{\Omega_n}\sin\left(\frac{\Omega_n t}{2}\right)\right]e^{i\Delta t/2} \\ C_{1,n+1} &= -i\,\frac{2g\sqrt{n+1}}{\Omega_n}\sin\left(\frac{\Omega_n t}{2}\right)e^{-i\Delta t/2} \\ \Omega_n &= (4g^2(n+1) + \Delta^2)^{1/2} \end{aligned}}$$

Quantized Light – Matter Interactions

$$\dot{c}_{1,n+1} = -ig\sqrt{n+1}\, e^{-i\Delta t} c_{2,n}$$
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Rabi Oscillations

$$P_2(n) = \cos^2\left(\frac{\Omega_n t}{2}\right) + \left(\frac{\Delta}{\Omega_n}\right)^2 \sin^2\left(\frac{\Omega_n t}{2}\right)$$

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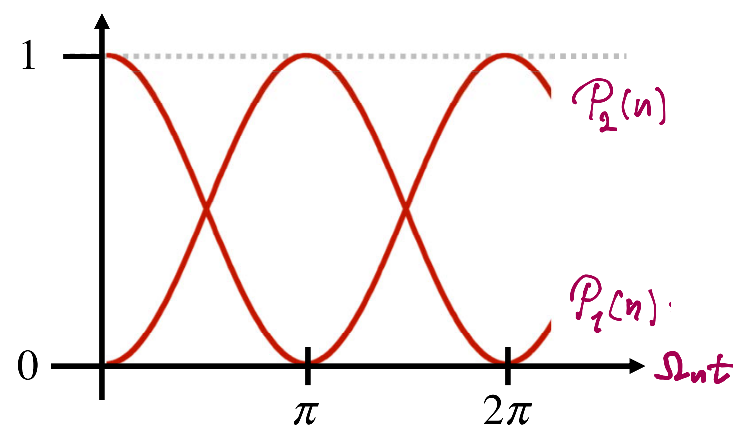
Example: $\Delta = 0$



Quantized Light – Matter Interactions

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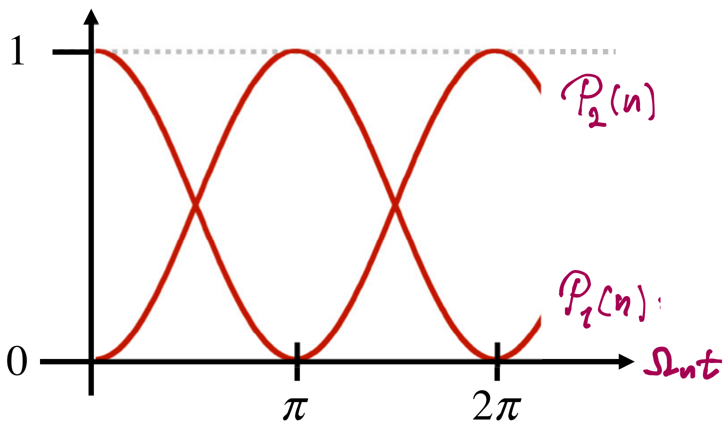

Vacuum Rabi Oscillations

If $|\psi(0)\rangle = |2,0\rangle$ ⇨ **no photons in field**

yet $|2,0\rangle$ **evolves into** $|1,1\rangle$

Uniquely QED phenomenon!

Asymmetry $\begin{cases} |2, n=0\rangle \longrightarrow |1, n=1\rangle \\ |1, n=0\rangle \longrightarrow |1, n=0\rangle \end{cases}$

holds germ of Spontaneous Decay

Quantized Light – Matter Interactions

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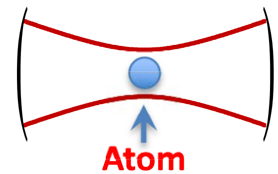
Next: More Cavity QED



2-level atom

Single cavity mode

What happens with a Coherent State in the Cavity mode?

(Quantum-Classical correspondence)

Initial atom-field state:

$$|\psi(0)\rangle = |1\rangle \otimes |\alpha\rangle = \sum_n C_n |1,n\rangle, \quad C_n = e^{-\frac{1}{2}|\alpha|^2} \frac{\alpha^n}{\sqrt{n!}}$$

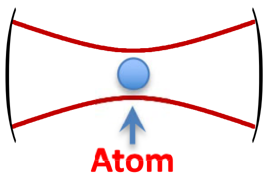
atom field

From Rabi solutions: $\Delta = 0$ ⇨

$$C_{1,n} = \cos\left(\frac{\Omega_n t}{2}\right), \quad C_{2,n-1} = -i \sin\left(\frac{\Omega_n t}{2}\right)$$

Quantized Light – Matter Interactions

Next: More Cavity QED



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Therefore uncoupled

$$\boxed{|\psi(t)\rangle = C_0|1,0\rangle + \sum_{n=1}^{\infty} C_n\left[\cos\left(\frac{\Omega_n t}{2}\right)|1,n\rangle - i\sin\left(\frac{\Omega_n t}{2}\right)|2,n-1\rangle\right]}$$

Consider the Atomic Excited State Population

$$\mathcal{P}_2(t) = \sum_{n=0}^{\infty} \mathcal{P}_{2,n} = \sum_{n=0}^{\infty} |\langle 2,n|\psi(t)\rangle|^2$$

$$= \sum_{n=0}^{\infty} |C_n|^2 \sin^2\left(\frac{\Omega_n t}{2}\right)$$

$$= \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!} e^{-|\alpha|^2} \sin^2\left(\frac{\Omega_n t}{2}\right)$$

Use $|\alpha|^2 = \bar{n}$ and $\Omega_n = 2g\sqrt{n}$

$$\boxed{\mathcal{P}_2(t) = \sum_{n=0}^{\infty} \frac{(\bar{n})^n e^{-\bar{n}}}{n!} \sin^2(g\sqrt{n}t)}$$

Quantized Light – Matter Interactions

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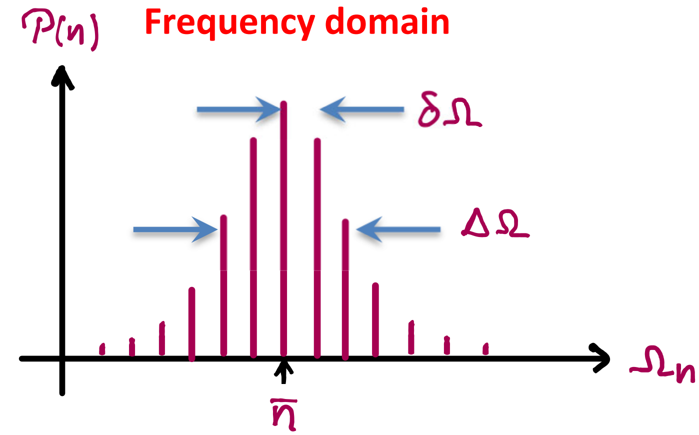
– **Poisson weighted average of sinusoids**

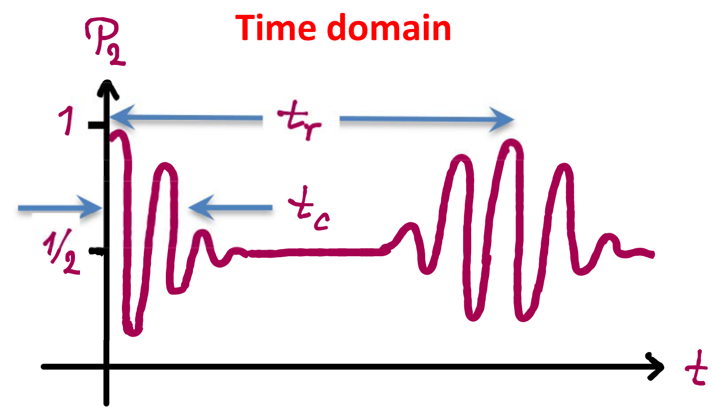
– **Sinusoids gradually dephase over time**

Collapse of oscillation amplitude

Frequency domain



Time domain



Quantized Light – Matter Interactions

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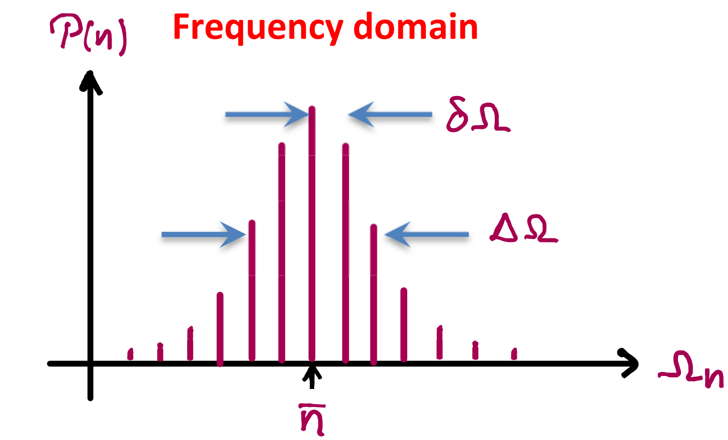
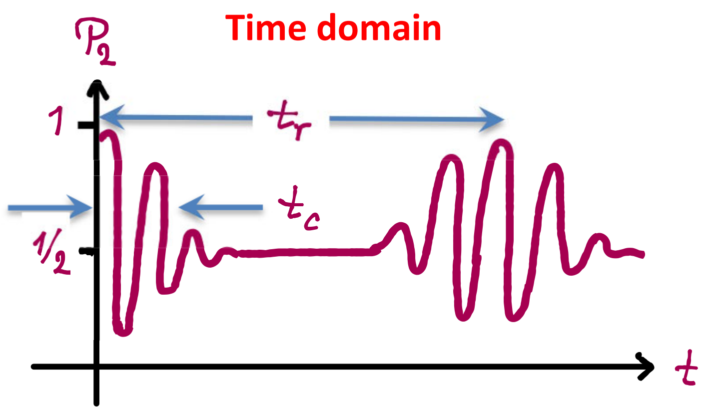
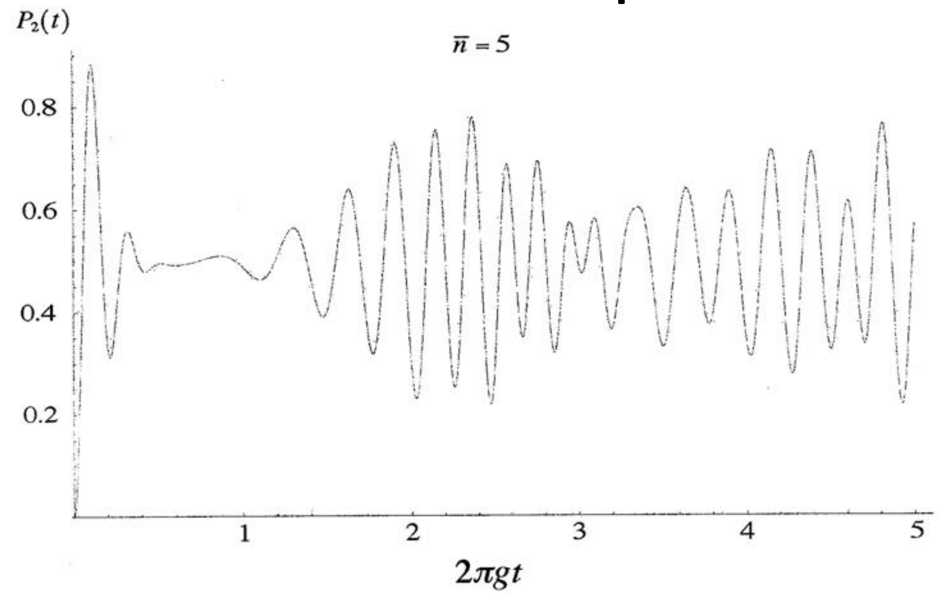
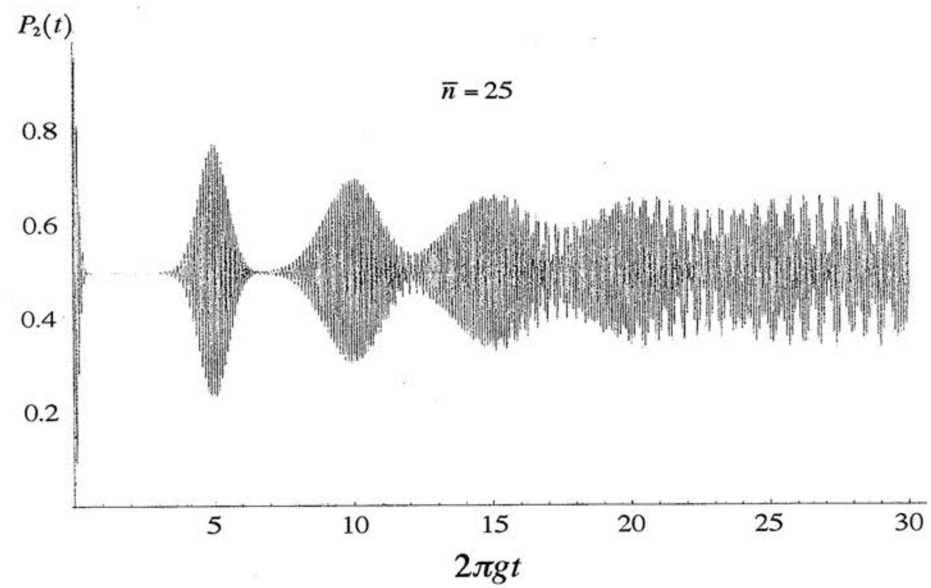




Numerical examples





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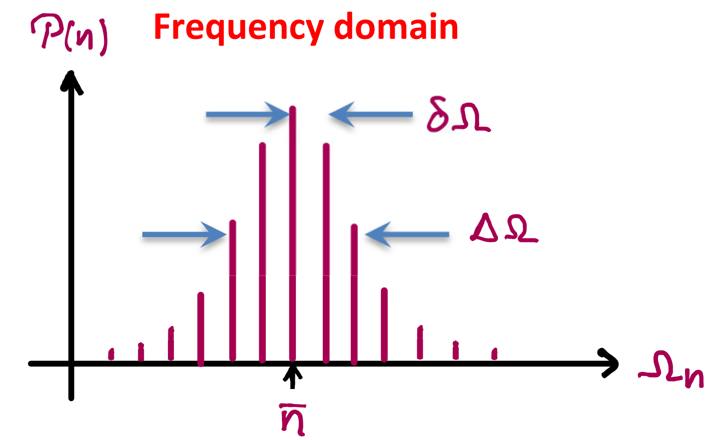

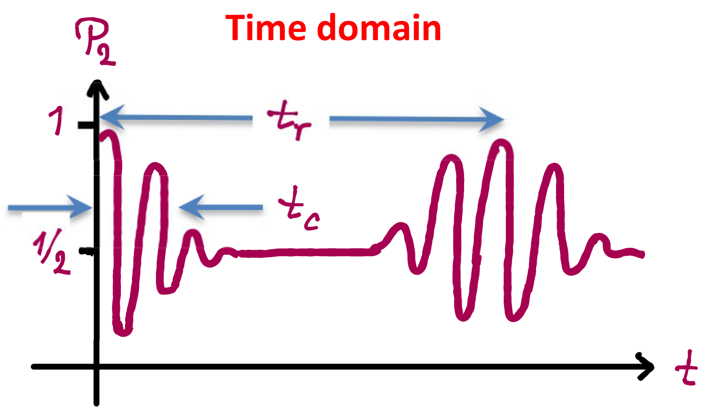

Use $\Delta n = \sqrt{\bar{n}}$ ➡ $\Delta\Omega \sim \Delta\Omega_{\bar{n}+\sqrt{\bar{n}}} - \Delta\Omega_{\bar{n}-\sqrt{\bar{n}}}$

$$\boxed{t_c = \frac{1}{\Delta\Omega} \sim \frac{1}{2g\sqrt{\bar{n}+\sqrt{\bar{n}}} - 2g\sqrt{\bar{n}-\sqrt{\bar{n}}}} \sim \frac{1}{2g}}$$

for $\bar{n} \gg \sqrt{\bar{n}}$

Rephasing: when $(\Omega_{\bar{n}} - \Omega_{\bar{n}-1})t_r \simeq 2\pi m$

Similar arguments -> Revival time

$$\boxed{t_r \sim \frac{2\pi}{\delta\Omega} \sim \frac{2\pi\sqrt{\bar{n}}}{g}}$$

Quantized Light – Matter Interactions

Use $\Delta n = \sqrt{\bar{n}}$ ⇨ $\Delta\Omega \sim \Delta\Omega_{\bar{n}+\sqrt{\bar{n}}} - \Delta\Omega_{\bar{n}-\sqrt{\bar{n}}}$

$$\boxed{t_c = \frac{1}{\Delta\Omega} \sim \frac{1}{2g\sqrt{\bar{n}+\sqrt{\bar{n}}} - 2g\sqrt{\bar{n}-\sqrt{\bar{n}}}} \sim \frac{1}{2g}}$$

for $\bar{n} \gg \sqrt{\bar{n}}$

Rephasing: when $(\Omega_{\bar{n}} - \Omega_{\bar{n}-1})t_r \simeq 2\pi m$

Similar arguments -> Revival time

$$\boxed{t_r \sim \frac{2\pi}{\delta\Omega} \sim \frac{2\pi\sqrt{\bar{n}}}{g}}$$

Collapse & Revival Dynamics

⇩

Pure Quantum Phenomenon

(“graininess” of photons)

Classical limit

$V \to \infty$ ⇨ $\mathcal{E}_k \to 0$ ⇨ $g \to 0$, $t_c \to \infty$

$\bar{n} \to \infty$ ⇨ $\frac{\Delta\Omega}{\Omega_{\bar{n}}} \to 0$ ⇨ $\Omega_{\bar{n}} > 0$

well defined

⇩

$$\Omega_{\bar{n}} = 2g\sqrt{\bar{n}} = \frac{\vec{\mu}_{21} \cdot 2\vec{\varepsilon}_k \mathcal{E}_k \sqrt{\bar{n}}}{\hbar} = \frac{\vec{\mu}_{21} \cdot \vec{E}}{\hbar}$$

Classical Rabi frequency

mean field $\langle \alpha(t) | \hat{\vec{E}} | \alpha(t) \rangle$

Quantized Light – Matter Interactions

Use $\Delta n = \sqrt{\bar{n}}$ $\Rightarrow$ $\Delta\Omega \sim \Delta\Omega_{\bar{n}+\sqrt{\bar{n}}} - \Delta\Omega_{\bar{n}-\sqrt{\bar{n}}}$

$$\boxed{t_c = \frac{1}{\Delta\Omega} \sim \frac{1}{2g\sqrt{\bar{n}+\sqrt{\bar{n}}} - 2g\sqrt{\bar{n}-\sqrt{\bar{n}}}} \sim \frac{1}{2g}}$$

for $\bar{n} \gg \sqrt{\bar{n}}$

Rephasing: when $(\Omega_{\bar{n}} - \Omega_{\bar{n}-1})t_r \simeq 2\pi m$

Similar arguments -> Revival time

$$\boxed{t_r \sim \frac{2\pi}{\delta\Omega} \sim \frac{2\pi\sqrt{\bar{n}}}{g}}$$

Collapse & Revival Dynamics

$\Downarrow$

Pure Quantum Phenomenon

("graininess" of photons)

Classical limit

$$\begin{cases} V \to \infty \Rightarrow \mathcal{E}_k \to 0 \Rightarrow g \to 0,\; t_c \to \infty \\ \bar{n} \to \infty \Rightarrow \dfrac{\Delta\Omega}{\Omega_{\bar{n}}} \to 0 \Rightarrow \Omega_{\bar{n}} > 0 \end{cases}$$

well defined

$\Downarrow$

$$\boxed{\Omega_{\bar{n}} = 2g\sqrt{\bar{n}} = \frac{\vec{\mu}_{21} \cdot 2\vec{\varepsilon}_k \mathcal{E}_k \sqrt{\bar{n}}}{\hbar} = \frac{\vec{\mu}_{21} \cdot \vec{E}}{\hbar}}$$

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