OPTI 544 **1st Midterm** **March 6, 2020**

I

Consider a gas of lithium atoms with a resonance frequency $\omega_{\text{Li}} = 3.10\times10^{15}\,\text{s}^{-1}$, trapped and cooled so they are stationary in the lab frame. The dipolar decay rate for the transition is $\beta = 3.68\times10^{7}\,\text{s}^{-1}$. An on-resonance linearly polarized laser beam tuned exactly on resonance, $\Delta = \omega_{\text{Li}} - \omega = 0$, propagates through this medium for a distance $L = 0.001\,\text{m}$, and the transmission is measured to be $T = 0.001$. In the following, use approximations appropriate for weakly polarizable media.

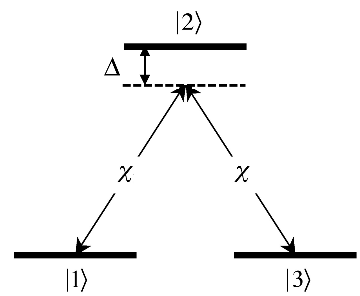
(a) Use the electron oscillator model to estimate the number density N of atoms. Your final answer must be a number. (20%)

(b) Find an expression for $n_{\text{R}}(\omega)$, the frequency dependent real index of refraction, in terms of $n_{\text{I}}(\omega)$, the imaginary index of refraction. (10%)

(c) Estimate the maximum value of the real index of refraction near resonance. Your final answer must be a number. (20%)

II

Consider Raman coupling in a 3-level atom as indicated in the figure. The ground-excited state transitions are driven by separate monochromatic fields with identical real-valued Rabi frequencies, $\chi_{12} = \chi_{32} = \chi$, and a common detuning Δ. The ground states are energy-degenerate and there is no Raman detuning, $\delta = 0$. Thus, in the rotating frame and rotating wave approximation, the Hamiltonian for the 3-level system is of the form shown.



$$H = \hbar\begin{pmatrix} 0 & \chi/2 & 0 \\ \chi/2 & \Delta & \chi/2 \\ 0 & \chi/2 & 0 \end{pmatrix}$$

(a) Write down the Schrödinger equation in terms of the density operator ρ and the Hamiltonian operator H. (10%)

(b) How many populations and coherences are present in ρ? Write out the separate differential equations governing the evolution of each. To ensure that you got the right answer, use any simple checks you can think of. (20%)

The excited state $|2\rangle$ decays spontaneously to the ground states $|1\rangle$ and $|3\rangle$ at rates $A = A_{21} + A_{31}$, where $A_{21} = A_{31} = A/2$.

(c) Following the general arguments used to arrive at the equivalent equations for a 2-level atom, add relaxation terms to the equations from (b) so as to account for the effects of spontaneous decay. For the various populations and coherences, explain in words why these terms are of the form you indicate. (20%)

Some numbers:	$c = 2.998\times10^{-8}\,\text{m/s}$	$e = 1.602\times10^{-19}\,\text{C}$
	$\varepsilon_0 = 8.854\times10^{-12}\,\text{F/m}$	$m_e = 9.109\times10^{-31}\,\text{kg}$
	$\mu_0 = 4\pi\times10^{-7}\,\text{N/A}^2$	$m_p = 1.672\times10^{-27}\,\text{kg}$
	$\hbar = 1.055\times10^{-37}\,\text{Js}$	$m_{\text{Li}} = 1.17\times10^{-26}\,\text{kg}$