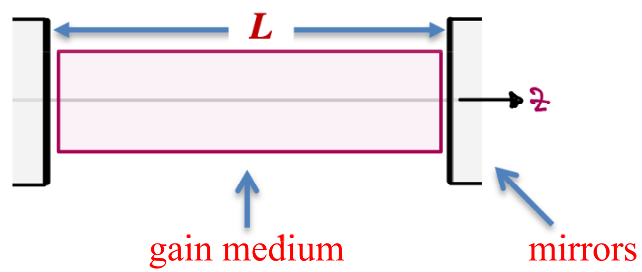
Semi-Classical Laser Theory

Begin 03-17-2021

Lasing action: requires a gain medium & feedback



(∗) As usual we simplify to focus on the key concepts ➡ 1D cavity

(∗) For spherical mirror resonators, see M&E Ch. 14

Optical Resonator/Cavity ➡

Eigenmodes of the Electromagnetic Field

Plane Parallel Mirrors ➡ standing waves

Length L ➡ wave number for m'th mode

$$k = \frac{m\pi}{L}, \quad m \text{ integer}$$

Field in the m'th mode

$$\boxed{\vec{E}_m(z,t) = \vec{\varepsilon}_m \mathcal{E}_m(t)\sin(k_m z)e^{-i\omega t}}$$

Slowly Varying Envelope

Note: Gain ➡ dispersion in cavity

$$\omega \neq k_m c = \omega_m$$

Laser freq. Vacuum mode freq.

Polarization density, m'th mode

$$\vec{P}_m(z,t) = \vec{\varepsilon}_m 2N\mu^* \rho_{21}^{(m)}(z,t)\sin(k_m z)e^{-i\omega t}$$

$$\vec{P}(z,t) = \sum_m \vec{P}_m(z,t)$$ ⟵ Total polarization density in all modes

Note: Saturation effects ➡ Mode cross-talk

Semi-Classical Laser Theory

Wave eq. in a Resonator

– mimic loss by including current $\vec{J} = \sigma \vec{E}$

finite conductivity

4th Maxwell Eq.: $\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \sigma \vec{E}$

Wave Eq. in resonator, with distributed loss

$$\left(\frac{\partial^2}{\partial z^2} - \frac{\kappa}{c^2}\frac{\partial}{\partial t} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{E} = \frac{1}{\varepsilon_0 c^2}\frac{\partial^2}{\partial t^2}\vec{P}$$

$\kappa = \sigma/\varepsilon_0$ ← Phenomenological loss constant (losses + output coupling)

units 1/s

Wave Eq. for m'th mode in the resonator

$$\left(\frac{\partial^2}{\partial z^2} - \frac{\kappa}{c^2}\frac{\partial}{\partial t} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{\varepsilon}_m \mathcal{E}_m(t)\sin(k_m z)e^{-i\omega t}$$

$$= \frac{\vec{\varepsilon}_m}{\varepsilon_0 c^2} 2N\mu^* \sin(k_m z)\frac{\partial^2}{\partial t^2}\left(\rho_{21}^{(m)}(t)e^{-i\omega t}\right)$$

Apply SVEA & resonant approx., $\omega - \omega_m \ll \omega$ (HW)

$$\left[-i(\omega-\omega_m) + \frac{\kappa}{2} + \frac{\partial}{\partial t}\right]\mathcal{E}_m(t) = \frac{i\omega}{\varepsilon_0} N\mu^* \rho_{21}^{(m)}(t)$$

Quasi-steady state solution:

Fast atomic response
High-Q cavity
$(\beta \gg \kappa)$

⇒ $\rho_{21}^{(m)}(t)$ **in S. S. given** $\mathcal{E}(t)$

We can adiabatically eliminate $\rho_{21}^{(m)}(t)$

by replacing w/ S.S value given $\mathcal{E}(t)$, ρ_{11} & ρ_{22}

$$\rho_{21}(t) = -\frac{i\mu\mathcal{E}(t)}{2\hbar}\frac{\beta - i\Delta}{\beta^2 + \Delta^2}(\rho_{22} - \rho_{11})$$

Note: We will allow for some external process that potentially creates a population inversion

Semi-Classical Laser Theory

Substitute in Equation for $\mathcal{E}_m(t)$

$$\left[-i(\omega-\omega_m)+\frac{\kappa}{2}+\frac{\partial}{\partial t}\right]\mathcal{E}_m(t) = \frac{N|\mu|^2\omega}{2\varepsilon_0\hbar}\frac{\beta-i\Delta}{\beta^2+\Delta^2}(\rho_{22}-\rho_{11})\mathcal{E}_m(t)$$

Let $N_1 = N\rho_{11}$, $N_2 = N\rho_{22}$ and define

$$g \equiv \frac{|\mu|^2\omega}{\varepsilon_0\hbar c}\frac{\beta}{\Delta^2+\beta^2}(N_2-N_1) = \sigma(\Delta)(N_2-N_1)$$ gain

$$\delta \equiv \frac{\Delta}{\beta}g = \frac{\Delta}{\beta}\sigma(\Delta)(N_2-N_1)$$ dispersion

Fundamental Eq. of Semiclassical Laser Theory

$$\boxed{\frac{\partial}{\partial t}\mathcal{E}_m(t) = \frac{1}{2}\left[-\kappa+2i(\omega-\omega_m)+c(g-i\delta)\right]\mathcal{E}_m(t)}$$

The FESLT gives us insight into

- (∗) Threshold behavior
- (∗) Laser intensity and power output
- (∗) Laser frequency and linewidth

Equation for Laser intensity $I \propto \mathcal{E}^*\mathcal{E}$

$$\frac{dI}{dt} \propto \frac{\partial \mathcal{E}_m^*(t)}{dt}\mathcal{E}_m(t) + c.c.$$
$$= \frac{1}{2}\left[-\kappa-2i(\omega-\omega_m)+c(g-i\delta)\right]|\mathcal{E}_m(t)|^2 + c.c.$$

$$\frac{dI}{dt} = (cg-\kappa)I \Rightarrow \begin{cases} g > g_t : \text{exponential growth} \\ g < g_t : \text{exponential decay} \end{cases}$$

We define $g_t = \sigma(\Delta)\Delta N_t$, $\Delta N_t = \dfrac{\kappa}{c\sigma(\Delta)}$

Semi-Classical Laser Theory

The FESLT gives us insight into

(*) Threshold behavior

(*) Laser intensity and power output

(*) Laser frequency and linewidth

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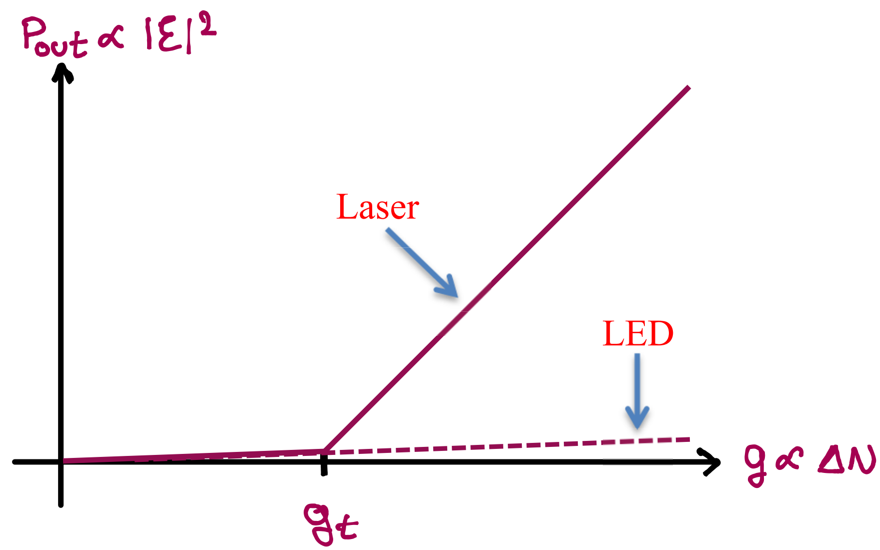
- $g > g_t$: exponential growth
- $g < g_t$: exponential decay

We define $g_t = \sigma(\Delta)\Delta N_t$, $\Delta N_t = \frac{\kappa}{c\sigma(\Delta)}$

These are Key parameters that characterizes a laser

$g_t = \frac{\kappa}{c}$	Threshold Gain
$\Delta N_t = \frac{\kappa}{c\sigma(\Delta)}$	Threshold Inversion

Example: Diode lasers & threshold behavior



End 03-17-2021

Semi-Classical Laser Theory

Begin 03-19-2021

Fundamental Eq. of Semiclassical Laser Theory

$$\frac{\partial \mathcal{E}_m(t)}{\partial t} = \frac{1}{2}\left[-\kappa + 2i(\omega-\omega_m) + c(g - i\delta)\right]\mathcal{E}_m(t)$$

$g \equiv \sigma(\Delta)(N_2 - N_1)$ — gain

$\delta \equiv \frac{\Delta}{\beta} g = \frac{\Delta}{\beta}\sigma(\Delta)(N_2-N_1)$ — dispersion

Laser Frequency in Steady State

Let $\frac{\partial}{\partial t}\mathcal{E}_m = 0$ and consider imaginary part of FESLT

$$i(\omega - \omega_m) - i\frac{c\delta}{2} = 0, \quad \text{with } \delta = \frac{\Delta}{\beta} g$$

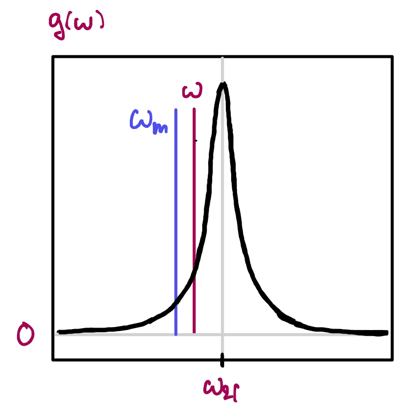
$$\omega_m - \omega = -\frac{gc}{2\beta}\Delta = \frac{gc}{2\beta}(\omega - \omega_{21})$$

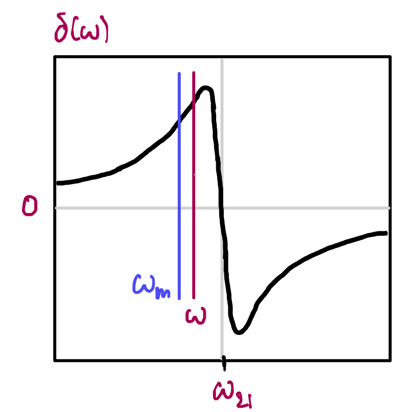
Solve for ω :

$$\omega = \frac{\omega_m + \frac{gc}{2\beta}\omega_{21}}{1 + gc/2\beta} \approx \omega_m + \frac{cg}{2\beta}(\omega_{21} - \omega_m)$$

laser frequency; for $\frac{gc}{2\beta} \ll 1$; frequency pulling

Physical interpretation – note $\delta(\omega) > 0$ for $\omega < \omega_{21}$





From MBE's $n_R = 1 - \frac{\delta\omega}{2k}$ ⇨ $n_R < 1$

⇨ Optical L < physical L ⇨ ω increases

– Laser frequency is pulled towards resonance

Semi-Classical Laser Theory

Solve for ω :

$$\omega = \frac{\omega_m + \frac{gc}{2\beta}\,\omega_{21}}{1 + \frac{gc}{2\beta}} \approx \omega_m + \frac{cg}{2\beta}\left(\omega_{21} - \omega_m\right)$$

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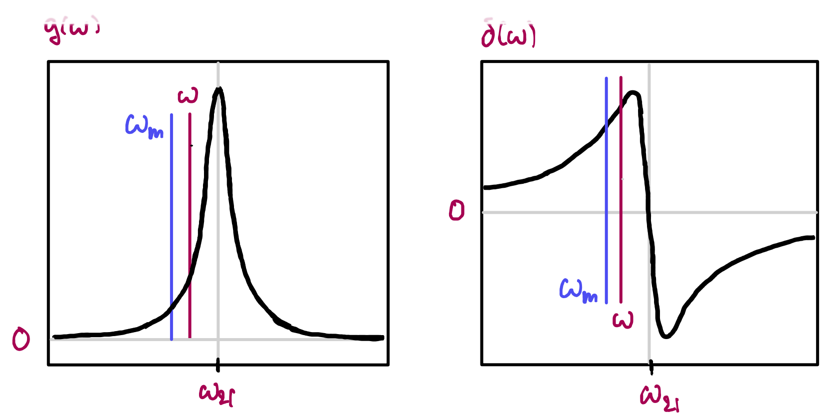

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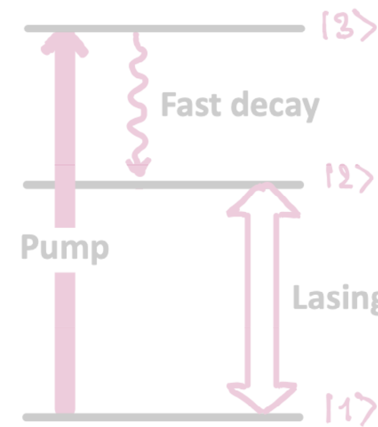
Gain requires Population Inversion ⇒

Laser Pumping Schemes

3-Level System

Ruby Laser

Hard to Pump!



4-Level System

Nd-YAG
Ti-Sappire
Er-Fiber (glass)
Organic Dye
Helium-Neon
Semiconductor

Easy to Pump!

Semi-Classical Laser Theory

Gain requires Population Inversion

Laser Pumping Schemes

3-Level System

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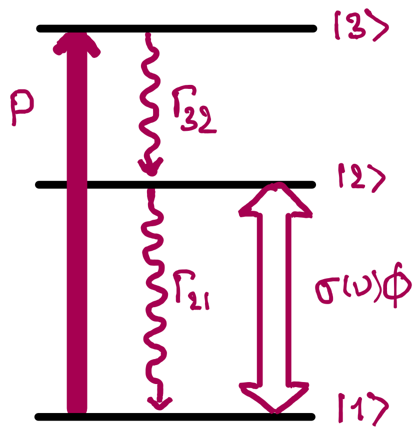

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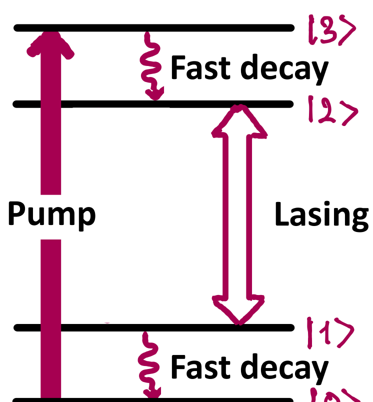
- Ti-Sappire
- Er-Fiber (glass)
- Organic Dye
- Helium-Neon
- Semiconductor

Easy to Pump!



Population Rate Equations – 3 level System

Let $\Gamma_{32} \gg P,\ \Gamma_{21},\ \sigma(\nu)\phi \;\Rightarrow\; N_3 \sim 0$

$$\boxed{\begin{aligned} \dot{N}_1 &= -PN_1 + \Gamma_{21}N_2 + \sigma(\nu)\phi\,(N_2 - N_1) \\ \dot{N}_2 &= PN_1 - \Gamma_{21}N_2 - \sigma(\nu)\phi\,(N_2 - N_1) \end{aligned}}$$

End 03-19-2021 Begin 03-22-2021

Steady State Solution (Home Work Set 6)

$$N_2 - N_1 = \frac{(P - \Gamma_{21})(N_2 + N_1)}{P + \Gamma_{21} + 2\sigma(\nu)\phi}$$

Use $\begin{cases} N_1 + N_2 = N \\ g(\nu) = \sigma(\nu)(N_2 - N_1) \end{cases}$

$$g(\nu) = \sigma(\nu)\frac{(P - \Gamma_{21})N}{P + \Gamma_{21} + 2\sigma(\nu)\phi} > 0 \quad \text{iff} \quad P > \Gamma_{21}$$

Semi-Classical Laser Theory

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Divide top & bottom w/ $P + \Gamma_{21}$

$$g(\nu) = \frac{g_0(\nu)}{1+\phi/\phi_{sat}}$$

Saturated Gain

$$g_0(\nu) = \sigma(\nu)\frac{(P-\Gamma_{21})N}{P+\Gamma_{21}}$$

Small Signal Gain

$$\phi_{sat} = \frac{P+\Gamma_{21}}{2\sigma(\nu)}$$

Saturation Flux

$$I_{sat} = h\nu\phi_{sat}$$

Saturation Intensity

Semi-Classical Laser Theory

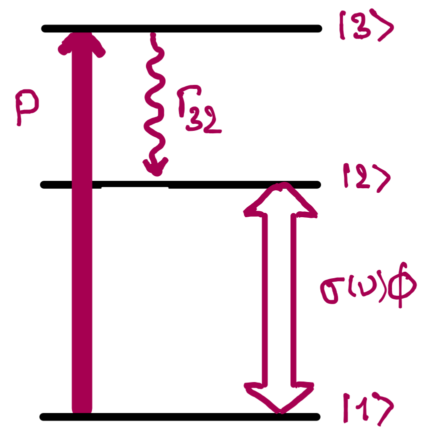
Gain requires Population Inversion ⇨

Laser Pumping Schemes

3-Level System

Ruby Laser

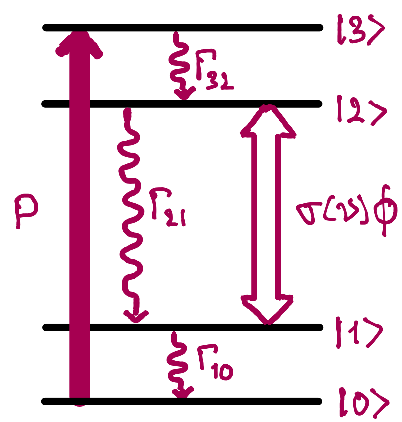
Hard to Pump!



4-Level System

Nd-YAG
Ti-Sappire
Er-Fiber (glass)
Organic Dye
Helium-Neon
Semiconductor

Easy to Pump!



Population Rate Equations – 4 level System

Let $\Gamma_{32} \gg P,\ \Gamma_{21},\ \sigma(\nu)\phi$ ⇨ $N_3 \sim 0$

$$\dot{N}_0 = -PN_0 + \Gamma_{10} N_1$$

$$\dot{N}_1 = -\Gamma_{10} N_1 + \Gamma_{21} N_2 + \sigma(\nu)\phi(N_2 - N_1)$$

$$\dot{N}_2 = PN_0 - \Gamma_{21} N_2 - \sigma(\nu)\phi(N_2 - N_1)$$

Steady State Solution (Home Work Set 6)

$$N_2 - N_1 = \frac{P(\Gamma_{10} - \Gamma_{21})N}{P(\Gamma_{10} + \Gamma_{21}) + \Gamma_{10}\Gamma_{21} + (2P + \Gamma_{10})\sigma\phi}$$

Semi-Classical Laser Theory

Population Rate Equations – 4 level System

Let $\Gamma_{32} \gg P,\ \Gamma_{21},\ \sigma(\nu)\phi \Rightarrow N_3 \sim 0$

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Steady State Solution (Home Work Set 6)

$$N_2 - N_1 = \frac{P(\Gamma_{10} - \Gamma_{21})N}{P(\Gamma_{10} + \Gamma_{21}) + \Gamma_{10}\Gamma_{21} + (2P + \Gamma_{10})\sigma\phi}$$

Divide top & bottom w/ $P(\Gamma_{10} + \Gamma_{21}) + \Gamma_{10}\Gamma_{21}$

$$g(\nu) = \frac{g_0(\nu)}{1 + \phi/\phi_{sat}}$$

Saturated Gain

$$g_0(\nu) = \frac{\sigma(\nu)P(\Gamma_{10} - \Gamma_{21})N}{P(\Gamma_{10} + \Gamma_{21}) + \Gamma_{10}\Gamma_{21}}$$

Small Signal Gain

$$\phi_{sat} = \frac{P(\Gamma_{10} + \Gamma_{21}) + \Gamma_{10}\Gamma_{21}}{2(P + \Gamma_{10})\sigma(\nu)}, \quad I_{sat} = h\nu\phi_{sat}$$

Saturation Flux

Saturation Intensity

Semi-Classical Laser Theory

Divide top & bottom w/ $P(\Gamma_{10}+\Gamma_{21})+\Gamma_{10}\Gamma_{21}$

$$g(\nu) = \frac{g_0(\nu)}{1+\phi/\phi_{sat}}$$

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Saturation Flux | **Saturation Intensity**

Threshold Inversion and Pumping Rates

Example: 3-level system: $\Delta N = \frac{(P-\Gamma_{21})N}{P+\Gamma_{21}+2\sigma\phi}$

By definition $g_t = \sigma(\nu)\Delta N_t$

Threshold Gain

Threshold Inversion

defines P_t

$$\Delta N_t = \frac{(P_t-\Gamma_{21})N}{P_t+\Gamma_{21}+\underbrace{2\sigma\phi}_{=0}} = \frac{(P_t-\Gamma_{21})N}{P_t+\Gamma_{21}}$$

below threshold

Solve for the Threshold Pumping Rate.

$$P_t^{3\text{-level}} = \frac{N+\Delta N_t}{N-\Delta N_t}$$

$$P_t^{4\text{-level}} = \frac{\Delta N_t}{N-\Delta N_t} \quad \text{for } \Gamma_{10} \gg \Gamma_{21}$$

Semi-Classical Laser Theory

Threshold Inversion and Pumping Rates

Example: 3-level system: $\Delta N = \frac{(P - \Gamma_{21})N}{P + \Gamma_{21} + 2\sigma\phi}$

By definition $g_t = \sigma(\nu)\Delta N_t$ ⇨ defines P_t

Threshold Gain

Threshold Inversion

$$\Delta N_t = \frac{(P_t - \Gamma_{21})N}{P_t + \Gamma_{21} + \underbrace{2\sigma\phi}_{=0}} = \frac{(P_t - \Gamma_{21})N}{P_t + \Gamma_{21}}$$

below threshold

Solve for the Threshold Pumping Rate.

$$P_t^{3\text{-level}} = \frac{N + \Delta N_t}{N - \Delta N_t}$$

$$P_t^{4\text{-level}} = \frac{\Delta N_t}{N - \Delta N_t} \quad \text{for} \quad \Gamma_{10} >> \Gamma_{21}$$

Gain under Lasing Conditions

Below threshold $g \leq g_t$ ⇨ $\begin{cases} \phi \leq \phi_{sat} \\ g(\nu) \sim g_0(\nu) \end{cases}$

Small Signal Gain

Above threshold: exp. growth of ϕ until the gain saturates, growth slows and stops

Steady State: $\boxed{g(\nu) = g_t = \kappa/c}$

Saturated Gain = Loss

Important Question:

– What if many modes see significant gain?

– It depends, and can be complicated !

Semi-Classical Laser Theory

Gain under Lasing Conditions

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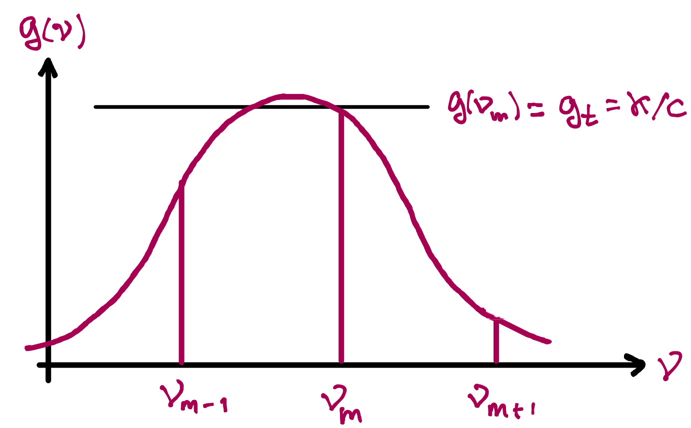
Homogeneous Gain Broadening

All atoms identical, couple identically to modes
(lifetime, collision broadening)

Consider a gradual increase in the pumping rate

$P \sim 0$ ➡ $P \sim P_t$ ➡ $P \gg P_t$



1st mode to reach threshold will lase, saturate gain, and clamp the inversion at its threshold value

Mode Competition
➡ Winner Takes All

End 03-22-2021

Begin 03-24-2021

Semi-Classical Laser Theory

Homogeneous Gain Broadening

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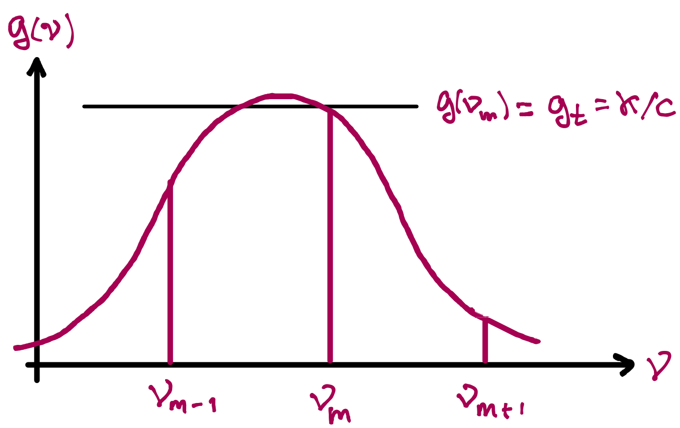


1st mode to reach threshold will lase, saturate gain, and clamp the inversion at its threshold value

Mode Competition
⇨ Winner Takes All

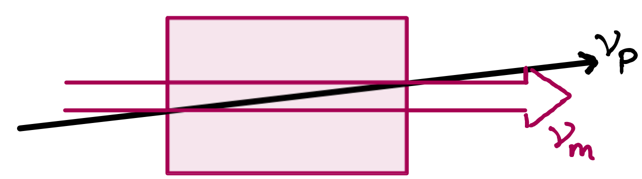
Inhomogeneous Gain Broadening

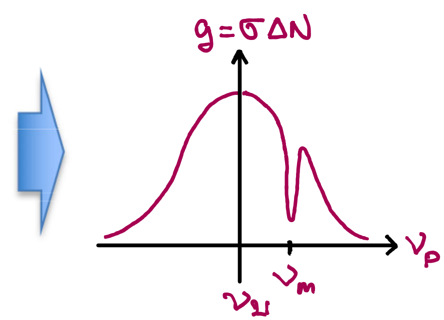
Atoms are different, couple differently to modes
(dopants in disordered host material, Doppler broadening in gas lasers)

We write the small signal gain in the medium as

$$g_0(\nu) = \sigma(\nu)\,\underbrace{\overbrace{2\beta S(\nu)}^{\text{normalized Line Shape}}\,\Delta N_0}_{\text{inversion available at freq. }\nu}$$

To observe saturation, we measure gain for a weak probe in the presence of a strong pump beam





The pump saturates the gain for atoms with transition freq. near ν_m only ⇨

Generally multi-mode Laser operation

Semi-Classical Laser Theory

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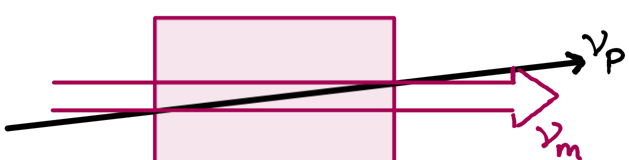

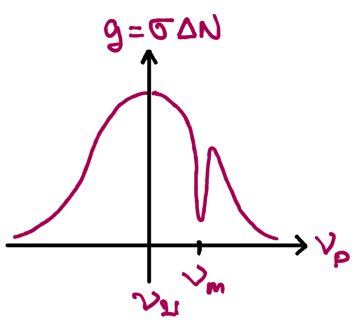

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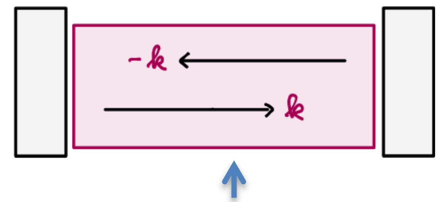
Generally multi-mode Laser operation

Spectral hole burning in gas lasers

Geometry:

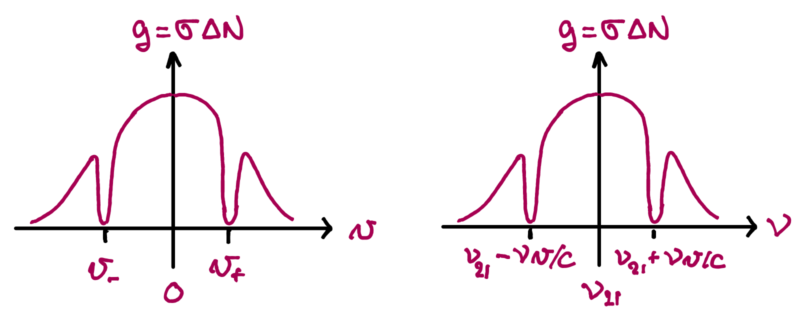


Doppler broadened gain medium

Due to Doppler shifts, each laser mode feeds on two velocity classes that are resonant for light traveling in opposite directions.

$$\left.\begin{aligned} \nu + \nu v/c &= \nu_{21} \\ \nu - \nu v/c &= \nu_{21} \end{aligned}\right\} \Rightarrow v_{\pm} = \pm(\nu_{21} - \nu)\, c/\nu$$

Resulting gain as function of velocity or frequency:



Semi-Classical Laser Theory

Spectral hole burning in gas lasers

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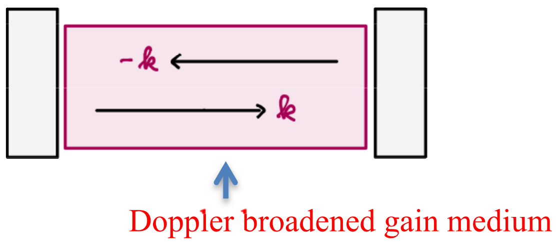

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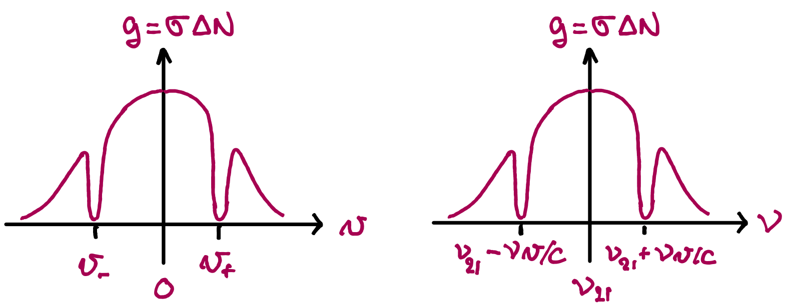


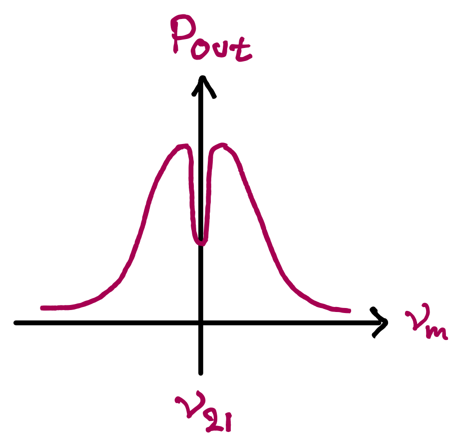
Lamb dip in gas lasers

(∗) Tune the resonator frequency ν_m towards the transition frequency ν_{21} of atoms at rest ⇨

(∗) The lasing mode feeds on increasingly large population classes ⇨ output power grows

(∗) When the spectral holes start overlapping the available population inversion decreases again ⇨ drop in output power centered on ν_{21}

(∗) This feature is known as the Lamb Dip



(∗) can be used for laser frequency stabilization

(∗) saturated absorption in vapor cells works better, yields better frequency stability

Semi-Classical Laser Theory

Lamb dip in gas lasers

(∗) Tune the resonator frequency ν_m towards the transition frequency ν_{21} of atoms at rest ⇨

(∗) The lasing mode feeds on increasingly large population classes ⇨ output power grows

(∗) When the spectral holes start overlapping the available population inversion decreases again ⇨ drop in output power centered on ν_{21}

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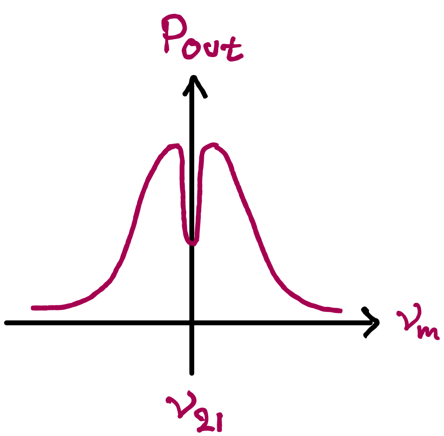

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Shawlow-Townes Formula for Laser Linewidth

Fundamental Question: What is the limit on the Laser linewidth (stability of the E-field phase)

Semiclassical Laser Theory ⇨ FESLT

(i) $$\frac{d\mathcal{E}}{dt} = \frac{1}{2}\left[-\kappa + 2i(\omega-\omega_m) + c(g - i\partial)\right]\mathcal{E}$$

(ii) $$\frac{\partial|\mathcal{E}|^2}{\partial t} = (cg-\kappa)|\mathcal{E}|^2$$

Eq. (ii) predicts a Steady State field amplitude

$$\frac{cg_0}{1+|\mathcal{E}_{ss}|^2/|\mathcal{E}|^2_{sat}} = \kappa \quad ⇨$$

$$|\mathcal{E}|^2_{ss} = \frac{cg_0-\kappa}{\kappa}|\mathcal{E}|^2_{sat} \sim \frac{cg_0}{\kappa}|\mathcal{E}|^2_{sat}$$

Far above threshold

Saturated Gain: $$g = \frac{g_0}{1+|\mathcal{E}|^2/|\mathcal{E}|^2_{sat}}$$

Semi-Classical Laser Theory

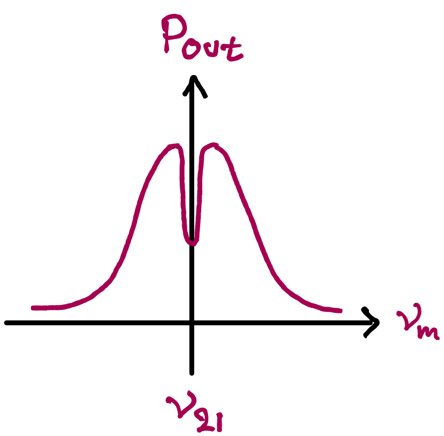
Lamb dip in gas lasers

(∗) Tune the resonator frequency ν_m towards the transition frequency ν_{21} of atoms at rest $\Rightarrow$

(∗) The lasing mode feeds on increasingly large population classes $\Rightarrow$ output power grows

(∗) When the spectral holes start overlapping the available population inversion decreases again $\Rightarrow$ drop in output power centered on ν_{21}

(∗) This feature is known as the Lamb Dip



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Far above threshold

(∗) A well designed Laser will relax back to $|\mathcal{E}|^2_{ss}$ after a perturbation

(∗) The field *Phase* is not determined by the FESLT

Semi-Classical Laser Theory

Shawlow-Townes Formula for Laser Linewidth

Fundamental Question: What is the limit on the Laser linewidth (stability of the E-field phase)

Semiclassical Laser Theory ➡ FESLT

$$\text{(i)}\quad \frac{d\mathcal{E}}{dt} = \frac{1}{2}\left[-\kappa + 2i(\omega - \omega_m) + C(g - i\delta)\right]\mathcal{E}$$

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Eq. (ii) predicts a Steady State field amplitude

$$\frac{Cg_0}{1+|\mathcal{E}|^2_{ss}/|\mathcal{E}|^2_{sat}} = \kappa \;\Rightarrow$$

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Far above threshold

(✱) **A well designed Laser will relax back to $|\mathcal{E}|^2_{ss}$ after a perturbation**

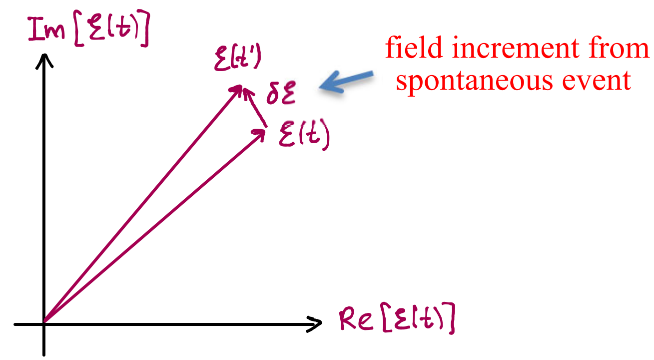
(✱) **The field *Phase* is not determined by the FESLT**

Beyond Semiclassical Laser Theory

(✱) **Eqs. (i) & (ii) accounts for absorption and stimulated emission**

(✱) ***Spontaneous* emission into the laser cavity is uncorrelated with the existing laser field $\mathcal{E}$**

Phasor representation of spontaneous emission

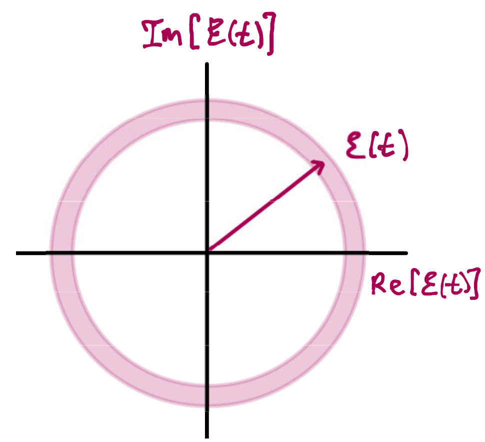


Amplitude relaxes back to $|\mathcal{E}|^2_{ss}$

Phase change remains

⬇

Phase diffusion



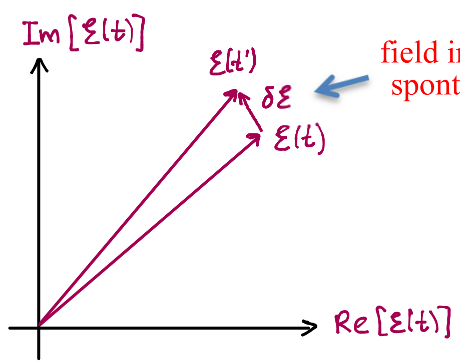
Semi-Classical Laser Theory

Beyond Semiclassical Laser Theory

(∗) Eqs. (i) & (ii) accounts for absorption and stimulated emission

(∗) *Spontaneous* emission into the laser cavity is uncorrelated with the existing laser field $\mathcal{E}$

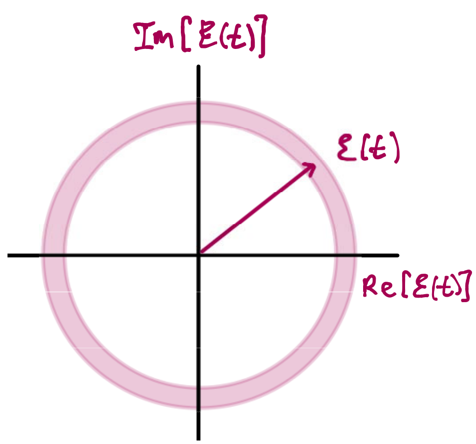
Phasor representation of spontaneous emission



Amplitude relaxes back to $|\mathcal{E}|^2_{ss}$

Phase change remains

Phase diffusion



We write the field as $\boxed{\mathcal{E}(t) \approx \mathcal{E}_0 e^{-i[\omega_0 t + \varphi(t)]}}$

phase diffusion

Key idea behind analysis:

The phase does a *Random Walk* $\left\{ \begin{array}{l} \text{step size } \Delta\varphi \\ \text{step rate } \gamma \end{array} \right.$

Note: Intuitively $\Delta\varphi \propto 1/|\mathcal{E}_{ss}|^2$ and $\gamma \propto N_2$

Statistical analysis of Random Walks:

(average distance walked in time τ)^2

$$\sim \Delta\phi(\tau)^2 = \gamma\tau\Delta\varphi^2 \equiv \frac{D}{2}\tau$$

Shawlow-Townes Formula

where the phase diffusion rate

$$\boxed{D = \frac{\kappa^2 \hbar\omega_0 N_2}{\Delta N_t P_{out}}}$$

Quantum Electrodynamics - QED

Introduction to Field Theory

(*) Question: How to develop a quantum theory for electromagnetic fields

(*) Answer: Develop a quantum theory for sound and use it as a source of inspiration.

(*) Note: We will make heavy use of classical Lagrange and Hamiltonian formalism. Check out Cohen-Tannoudji Vol. 2, Appendix III, Sections 1-3.