Maxwell-Bloch Equations

Begin lecture 03-15-2021

This gives us our final equation for the envelope:

$$\left(\frac{\partial}{\partial z}+\frac{1}{c}\frac{\partial}{\partial t}\right)\mathcal{E}(z,t)=\frac{ik}{\varepsilon_0}N\mu^*\rho_{21}(z,t)$$

where $\mu^*=\vec{\mu}_{12}\cdot\vec{\varepsilon}^*$

Write ρ_{21} in terms of the Bloch variables to get the

Maxwell-Bloch Equations

$$\left(\frac{\partial}{\partial z}+\frac{1}{c}\frac{\partial}{\partial t}\right)\mathcal{E}(z,t)=\frac{ik}{2\varepsilon_0}N\mu^*(u-iv)$$

$$\dot{u}=-\beta u+\mathrm{Im}[\chi]w+\Delta v$$

$$\dot{v}=-\beta v+\Delta u+\mathrm{Re}[\chi]w$$

$$\dot{w}=-\frac{1}{T_1}(1+w)-\mathrm{Re}[\chi]v-\mathrm{Im}[\chi]u$$

Note: The Maxwell-Bloch Equations are a key result. They lead to rich physics, including absorption, gain, dispersion, self-induced transparency, solitons, lasers, and much more.

End lecture 03-08-2021

Steady-State Solutions to MBE's

Steady state means that

$$\frac{1}{c}\frac{\partial\mathcal{E}}{\partial t}=0 \quad \& \quad \rho_{21}(z,t)\rightarrow\rho_{21}(\infty)=\frac{-i\chi/2}{\beta+i\Delta}(\rho_{22}-\rho_{11})$$

Combine with $\chi=\vec{\mu}_{21}\cdot\vec{\varepsilon}\,\mathcal{E}/\hbar=\mu\mathcal{E}/\hbar$

$$\frac{\partial\mathcal{E}}{\partial z}=\frac{ik}{\varepsilon_0}N\mu^*\left(\frac{-i\mu\mathcal{E}}{2\hbar}\right)\frac{1}{\beta+i\Delta}(\rho_{22}-\rho_{11})$$

$$=\frac{kN|\mu|^2}{2\hbar\varepsilon_0}\frac{\beta-i\Delta}{\Delta^2+\beta^2}(\rho_{22}-\rho_{11})\mathcal{E}$$

We can rewrite this as

$$\frac{\partial\mathcal{E}}{\partial z}=\frac{1}{2}(a-i\delta)w\mathcal{E}$$

$$a=\frac{kN|\mu|^2}{2\hbar\varepsilon_0}\frac{\beta}{\Delta^2+\beta^2}=N\sigma(\Delta)$$

$$\delta=\frac{kN|\mu|^2}{2\hbar\varepsilon_0}\frac{\Delta}{\Delta^2+\beta^2}=N\frac{\Delta}{\beta}\sigma(\Delta)$$

Maxwell-Bloch Equations

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$$\frac{1}{c}\frac{\partial \mathcal{E}}{\partial t} = 0 \quad \& \quad \rho_{21}(z,t) \to \rho_{21}(\infty) = \frac{-i\chi/2}{\beta + i\Delta}(\rho_{22} - \rho_{11})$$

Combine with $\chi = \vec{\mu}_{21}\cdot\vec{\varepsilon}\,\mathcal{E}/\hbar = \mu\mathcal{E}/\hbar$

$$\frac{\partial \mathcal{E}}{\partial z} = \frac{ik}{\varepsilon_0} N \mu^* \left(\frac{-i\mu\mathcal{E}}{2\hbar}\right)\frac{1}{\beta+i\Delta}(\rho_{22}-\rho_{11})$$

$$= \frac{kN|\mu|^2}{2\hbar\varepsilon_0}\frac{\beta - i\Delta}{\Delta^2+\beta^2}(\rho_{22}-\rho_{11})\mathcal{E}$$

We can rewrite this as

$$\boxed{\begin{aligned} \frac{\partial \mathcal{E}}{\partial z} &= \frac{1}{2}(a - i\delta)\,\omega\,\mathcal{E} \\ a &= \frac{kN|\mu|^2}{2\hbar\varepsilon_0}\frac{\beta}{\Delta^2+\beta^2} = N\sigma(\Delta) \\ \delta &= \frac{kN|\mu|^2}{2\hbar\varepsilon_0}\frac{\Delta}{\Delta^2+\beta^2} = N\frac{\Delta}{\beta}\sigma(\Delta) \end{aligned}}$$

To compare with our classical theory of dispersion, we solve for $\mathcal{E}(z)$ and plug into eq. for a plane wave.

Envelope: $\mathcal{E}(z) = \mathcal{E}_0 e^{\left(\frac{a\omega}{2}\right)z} e^{i\left(-\frac{\delta\omega}{2}\right)z}$

Field: $E(z) = \mathcal{E}(z)e^{ikz} = \mathcal{E}_0 e^{-n_I kz} e^{i n_R kz}$

Real & Imaginary Index of Refraction

$$\boxed{\begin{aligned} n_I &= -\frac{a\omega}{2k} = -\frac{N\omega}{2k}\sigma(\Delta) \\ n_R &= 1 - \frac{\delta\omega}{2k} = 1 - \frac{\Delta}{\beta}\frac{N\omega}{2k}\sigma(\Delta) \end{aligned}}$$

Analogous to results from Electron Oscillator

$$n_I(\omega) = \frac{Ne^2}{4\varepsilon_0 m_e \omega}\cdot\frac{\beta}{\Delta^2+\beta^2}, \quad n_R(\omega) = \frac{\Delta}{\beta}n_I(\omega)$$

Maxwell-Bloch Equations

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Behavior of the Intensity

$$\frac{\partial}{\partial z}[\mathcal{E}^*\mathcal{E}] = \mathcal{E}^*\frac{\partial \mathcal{E}}{\partial z} + \frac{\partial \mathcal{E}^*}{\partial z}\mathcal{E}$$

$$= \frac{1}{2}(a-i\delta)\omega|\mathcal{E}|^2 + \frac{1}{2}(a+i\delta)\omega|\mathcal{E}|^2 = a\omega|\mathcal{E}|^2$$

$$\boxed{\frac{\partial I}{\partial z} = a\omega I \simeq a(\rho_{22}-\rho_{11}) I}$$

Note that $\begin{cases} a = N\sigma(\Delta) \geq 0 \\ I(z) = I(0)\, e^{a(\rho_{22}-\rho_{11})z} \end{cases}$

Exp. Decay of I for $\rho_{22}-\rho_{11} < 0$

Exp. growth of I for $\rho_{22}-\rho_{11} > 0$

must be maintained by some external process

Maxwell-Bloch Equations

Behavior of the Intensity

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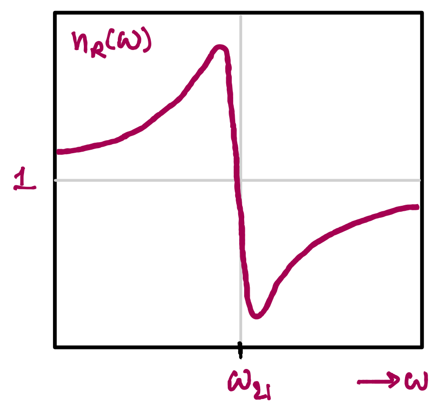
Behavior of the Dispersion:

Real & Imaginary Index of Refraction

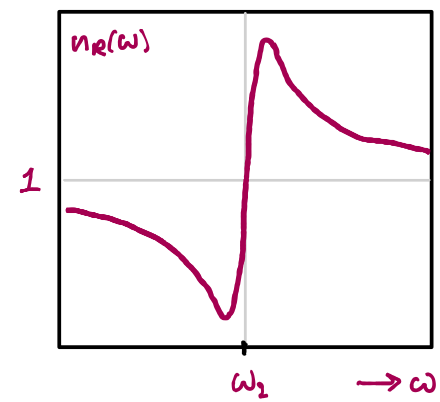
$$n_I = -\frac{a w}{2k} = -\frac{N w}{2k}\sigma(\Delta)$$

$$n_R = 1 - \frac{\delta w}{2k} = 1 - \frac{\Delta}{\beta}\frac{N w}{2k}\sigma(\Delta)$$

$w<1$ **absorption**



$w>1$ **gain**



Maxwell-Bloch Equations

Behavior of the Dispersion:

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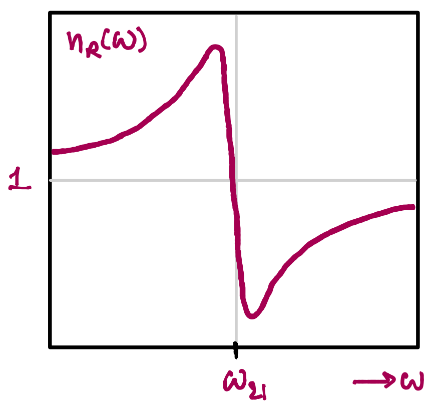
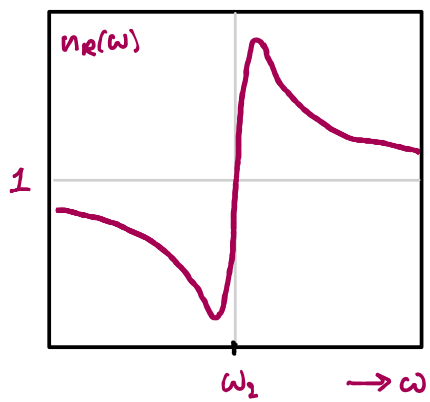


$w>1$ gain



End lecture 03-08-2021

Self-Induced Transparency & Solitons

(*) Example of a non-trivial application of the MBE's in the context of pulse propagation (highly dynamic, non-steady state behavior.

(*) The pulse area theorem suggests a light pulse with the proper envelope will act as a 2π pulse. Thus, if the pulse is shorter than the excited state lifetime it may propagate without loss. Correct shaping may also allow propagation without changes in pulse shape.

(*) See Lecture Notes, Slusher & Gibbs 1972.

Envelope: $\mathcal{E}(z,t) = \frac{2\hbar}{\mu\tau}\,\mathrm{sech}(\xi/\tau)$, $\xi = t - z/v$, $\Delta = 0$

$\Rightarrow$ $\chi(z,t) = \frac{2}{\tau}\,\mathrm{sech}(\xi/\tau)$, $\theta = \int_{-\infty}^{\infty}\chi(\xi/\tau)\,dt = 2\pi$

Self-consistent solution with the the properties of a Soliton

$$\mathcal{E}(z,t) = \frac{2\hbar}{\mu\tau}\,\mathrm{sech}(\xi/\tau)$$

$$u(\xi/\tau) = 0$$

$$v(\xi/\tau) = 2\,\mathrm{sech}(\xi/\tau)\tanh(\xi/\tau)$$

$$w(\xi/\tau) = -1 + 2\,\mathrm{sech}(\xi/\tau)$$

Maxwell-Bloch Equations

Self-Induced Transparency & Solitons

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In the SVEA version of the Wave Eq.

$$\left(\frac{\partial}{\partial z} + \frac{1}{c}\frac{\partial}{\partial t}\right)\mathcal{E}(z,t) = \frac{ik}{\varepsilon_0} N\mu^*(u - iv)$$

Substitute solutions for $\mathcal{E}$, u and v to get

$$\frac{2\hbar}{\mu\tau}\left(\frac{\partial}{\partial z} + \frac{1}{c}\frac{\partial}{\partial z}\right)\mathrm{sech}\left(\frac{t - z/V}{\tau}\right) =$$

$$\frac{2\hbar}{\mu\tau}\left(\frac{-1}{V\tau} + \frac{1}{c\tau}\right)\left[-\mathrm{sech}\left(\frac{t - z/V}{\tau}\right)\tanh\left(\frac{t - z/V}{\tau}\right)\right] =$$

$$\frac{2kN\mu^*}{\varepsilon_0}\,\mathrm{sech}\left(\frac{t - z/V}{\tau}\right)\tanh\left(\frac{t - z/V}{\tau}\right)$$

Solve for c/V to get

$$\frac{c}{V} = 1 + \frac{kN|\mu|^2}{2\varepsilon_0\hbar}c\tau^2 = 1 + \tfrac{1}{2}\alpha\beta c\tau^2$$

where $\alpha = \frac{kN|\mu|^2}{2\varepsilon_0\hbar\beta} = N\sigma(0)$ (on-resonance absorption coeff.)

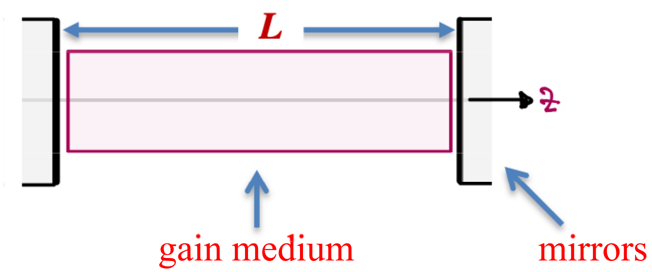
Consider Na vapor, $\lambda = 589\,nm$, $N = 10^{18} m^{-3}$, $T \sim 0K$, and $\beta = 2\pi \times 4.9\,MHz$ (completely opaque on res.)

Assuming $V \sim \frac{1}{2}c \Rightarrow \frac{c}{V} \sim 2 = 1 + \alpha\beta c\tau^2 \Rightarrow \frac{1}{2}\alpha\beta c\tau^2 \sim 1$

we must have $\tau \sim \sqrt{\frac{2}{\alpha\beta c}} \sim 36\,ps \ll 16\,ns$!!

Semi-Classical Laser Theory

Lasing action: requires a gain medium & feedback



(∗) As usual we simplify to focus on the key concepts ➡ 1D cavity

(∗) For spherical mirror resonators, see M&E Ch. 14

Optical Resonator/Cavity ➡

Eigenmodes of the Electromagnetic Field

Plane Parallel Mirrors ➡ standing waves

Length L ➡ wave number for m'th mode

$$k = \frac{m\pi}{L}, \quad m \text{ integer}$$

Field in the m'th mode

$$\boxed{\vec{E}_m(z,t) = \vec{\varepsilon}_m \mathcal{E}_m(t) \sin(k_m z) e^{-i\omega t}}$$

Slowly Varying Envelope

Note: Gain ➡ dispersion in cavity

$$\omega \neq k_m c = \omega_m$$

Laser freq. Vacuum mode freq.

Polarization density, m'th mode

$$\vec{P}_m(z,t) = \vec{\varepsilon}_m 2N\mu^* \rho_{21}^{(m)}(z,t) \sin(k_m z) e^{-i\omega t}$$

$$\vec{P}(z,t) = \sum_m \vec{P}_m(z,t) \longleftarrow \text{Total polarization density in all modes}$$

Note: Saturation effects ➡ Mode cross-talk

Semi-Classical Laser Theory

Field in the $m'th$ mode

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Wave eq. in a Resonator

– mimic loss by including current $\vec{J} = \sigma\vec{E}$

finite conductivity

4th Maxwell Eq.: $\nabla\times\vec{H} = \frac{\partial\vec{D}}{\partial t} + \sigma\vec{E}$

Wave Eq. in resonator, with distributed loss

$$\left(\frac{\partial^2}{\partial z^2} - \frac{\kappa}{c^2}\frac{\partial}{\partial t} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{E} = \frac{1}{\varepsilon_0 c^2}\frac{\partial^2}{\partial t^2}\vec{P}$$

$$\kappa = \sigma/\varepsilon_0 \leftarrow$$

Phenomenological loss constant (losses + output coupling)

units 1/s

Wave Eq. for $m'th$ mode in the resonator

$$\left(\frac{\partial^2}{\partial z^2} - \frac{\kappa}{c^2}\frac{\partial}{\partial t} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{\varepsilon}_m \mathcal{E}_m(t)\sin(k_m z)e^{-i\omega t}$$

$$= \frac{\vec{\varepsilon}_m}{\varepsilon_0 c^2} 2N\mu^*\sin(k_m z)\frac{\partial^2}{\partial t^2}\left(\rho_{21}^{(m)}(t)e^{-i\omega t}\right)$$

Semi-Classical Laser Theory

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finite conductivity

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$\kappa = \sigma/\varepsilon_0$ ← Phenomenological loss constant (losses + output coupling)

units 1/s

Wave Eq. for $m'th$ **mode in the resonator**

$$\left(\frac{\partial^2}{\partial z^2} - \frac{\kappa}{c^2}\frac{\partial}{\partial t} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{\mathcal{E}}_m \mathcal{E}_m(t)\sin(k_m z)e^{-i\omega t}$$

$$= \frac{\vec{\mathcal{E}}_m}{\varepsilon_0 c^2} 2N\mu^* \sin(k_m z)\frac{\partial^2}{\partial t^2}\left(\rho_{21}^{(m)}(t)e^{-i\omega t}\right)$$

Apply $SVEA$ **& resonant approx.,** $\omega - \omega_m \ll \omega$ **(HW)**

$$\left[-i(\omega-\omega_m) + \frac{\kappa}{2} + \frac{\partial}{\partial t}\right]\mathcal{E}_m(t) = \frac{i\omega}{\varepsilon_0} N\mu^* \rho_{21}^{(m)}(t)$$

Quasi-steady state solution:

Fast atomic response
High-Q cavity
$(\beta \gg \kappa)$
} ⇒ $\rho_{21}^{(m)}(t)$ in S. S. given $\mathcal{E}(t)$

We can adiabatically eliminate $\rho_{21}^{(m)}(t)$

by replacing w/ S.S value given $\mathcal{E}(t)$, ρ_{11} & ρ_{22}

$$\rho_{21}(t) = -\frac{i\mu\mathcal{E}(t)}{2\hbar}\frac{\beta - i\Delta}{\beta^2+\Delta^2}(\rho_{22}-\rho_{11})$$

Note: We will allow for some external process that potentially creates a population inversion

Semi-Classical Laser Theory

Apply $SVEA$ & resonant approx., $\omega - \omega_m \ll \omega$ (HW)

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Substitute in Equation for $\mathcal{E}_m(t)$

$$\left[-i(\omega-\omega_m) + \frac{\kappa}{2} + \frac{\partial}{\partial t}\right]\mathcal{E}_m(t) = \frac{N|\mu|^2\omega}{2\varepsilon_0\hbar}\frac{\beta - i\Delta}{\beta^2+\Delta^2}(\rho_{22}-\rho_{11})\mathcal{E}_m(t)$$

Let $N_1 = N\rho_{11}$, $N_2 = N\rho_{22}$ and define

$$g \equiv \frac{|\mu|^2\omega}{\varepsilon_0\hbar c}\frac{\beta}{\Delta^2+\beta^2}(N_2-N_1) = \sigma(\Delta)(N_2-N_1) \quad \text{gain}$$

$$\delta \equiv \frac{\Delta}{\beta} g = \frac{\Delta}{\beta}\sigma(\Delta)(N_2-N_1) \quad \text{dispersion}$$

Fundamental Eq. of Semiclassical Laser Theory

$$\boxed{\frac{\partial}{\partial t}\mathcal{E}_m(t) = \frac{1}{2}\left[-\kappa + 2i(\omega-\omega_m) + c(g-i\delta)\right]\mathcal{E}_m(t)}$$

Semi-Classical Laser Theory

Substitute in Equation for $\mathcal{E}_m(t)$

$$\left[-i(\omega-\omega_m)+\frac{\kappa}{2}+\frac{\partial}{\partial t}\right]\mathcal{E}_m(t) = \frac{N|\mu|^2\omega}{2\varepsilon_0\hbar}\frac{\beta-i\Delta}{\beta^2+\Delta^2}(\rho_{22}-\rho_{11})\mathcal{E}_m(t)$$

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The FESLT gives us insight into

(*) Threshold behavior

(*) Laser intensity and power output

(*) Laser frequency and linewidth

Equation for Laser intensity $I \propto \mathcal{E}^*\mathcal{E}$

$$\frac{dI}{dt} \propto \frac{\partial \mathcal{E}_m^*(t)}{dt}\mathcal{E}_m(t) + C.C.$$

$$= \frac{1}{2}\left[-\kappa-2i(\omega-\omega_m)+c(g-i\delta)\right]|\mathcal{E}_m(t)|^2 + C.C.$$

$$\frac{dI}{dt} = (cg-\kappa)I \Rightarrow \begin{cases} g > g_t : \text{exponential growth} \\ g < g_t : \text{exponential decay} \end{cases}$$

We define $g_t = \sigma(\Delta)\Delta N_t$, $\Delta N_t = \dfrac{\kappa}{c\sigma(\Delta)}$

Semi-Classical Laser Theory

The FESLT gives us insight into

- (∗) Threshold behavior
- (∗) Laser intensity and power output
- (∗) Laser frequency and linewidth

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We define $g_t = \sigma(\Delta)\Delta N_t$, $\Delta N_t = \frac{\kappa}{c\sigma(\Delta)}$

These are Key parameters that characterizes a laser

$g_t = \frac{\kappa}{c}$	Threshold Gain
$\Delta N_t = \frac{\kappa}{c\sigma(\Delta)}$	Threshold Inversion

Example: Diode lasers & threshold behavior