OPTI 544 1st Midterm Exam, March 12, 2021

Problem 1

A 2-level atom is in the ground state $|1\rangle$ at $t=0$. It is driven by a light field with a real-valued Rabi frequency χ and detuning $\Delta=0$. We assume there are no decays, collisions, or other relaxation processes.

(a) Given the above, write out (do not derive) the simplest possible expressions for the time dependent probability amplitudes $c_1(t)$ and $c_2(t)$ for the ground and excited states. (15%)

(b) Next, assume $|\psi(t=0)\rangle = (|1\rangle + i|2\rangle)/\sqrt{2}$. Modify the equations in (a) to match this new boundary condition. Then make a plot showing the populations of the ground and excited states as a function of time. (20%)

Hint: No lengthy math required.

(c) Finally, we assume the initial state is mixed, $\rho(t=0)=\frac{1}{2}(|1\rangle\langle 1|+|2\rangle\langle 2|)$. Make a plot of the time dependent populations, similar to the one in (b). Explain the reason for any differences. (15%)

Problem 2

We consider in the following the propagation of light at frequency ω through a medium of two-level atoms with transition frequency $\omega_{21} \simeq 1\times 10^{15}/\mathrm{s}$ $(\lambda_0 \simeq 2\,\mu\mathrm{m})$. For simplicity we assume the atoms are stationary and free from collisions, with a dipolar decay rate $\beta = 10^7/\mathrm{s}$.

(a) Based on the information given above, calculate the saturation flux and saturation intensity. (10%)

Next, consider the steady state propagation of light through the medium, assuming the number density of atoms is $N=10^{14}/\mathrm{m}^3$, the optical path length is $L=0.5\,\mathrm{m}$, and the light intensity is low enough that we can ignore saturation effects.

(b) Write out (do not derive) an expression for the detuning-dependent absorption coefficient a and transmission T. Then sketch both a and T as function of Δ/β, paying close attention to the shape of the curves near resonance.

Hint: Calculate $T(\Delta/\beta)$ for a reasonable number of samples before sketching. (20%)

(c) If your goal is to measure the transition frequency ω_{21} by sweeping the light frequency ω across resonance and looking for the minimum in $T(\Delta/\beta)$, what would you change to improve your spectroscopic resolution. (20%)