Free Electrons

Consider the limit $\omega >> \omega_0$, corresponding to effectively unbound electrons. This is a reasonable model of plasmas and metals.

We now have

$$\alpha(\omega) = \frac{e^2/m}{\omega_0^2 - \omega^2 - 2i\beta\omega} \approx -\frac{e^2}{m\omega^2} \Rightarrow$$

$$n(\omega) = \sqrt{1 + \frac{N\alpha(\omega)}{\varepsilon_0}} \approx \sqrt{1 - \frac{Ne^2}{\varepsilon_0 m\omega^2}} \equiv \sqrt{1 - \frac{\omega_P^2}{\omega^2}}$$

Here we have introduced the plasma frequency

$$\boxed{\omega_P = \sqrt{\frac{Ne^2}{\varepsilon_0 m}}}$$

Let $\left.\begin{array}{c}\omega_0 << \omega << \omega_P \\ |\omega_0 - \omega| >> \beta\end{array}\right\} \Rightarrow$ $n(\omega)$ purely imaginary, but no loss!

We now have

$$\mathbf{E}(z,t) = \hat{\varepsilon}E_0 e^{-i\omega[t - n(\omega)z/c]} = \hat{\varepsilon}E_0 e^{-i\omega t} e^{i\sqrt{\omega^2 - \omega_P^2}\,z/c}$$
$$= \hat{\varepsilon}E_0 e^{-i\omega t} e^{-b(\omega)z}; \qquad b(\omega) = -i\sqrt{\omega^2 - \omega_P^2}\Big/c,$$

where $b(\omega)$ is real-valued and positive.

Note: This result shows that the wave is not propagating, yet there is no loss. The implication is that a wave traveling through vacuum will be reflected at the boundary of a medium of this type. The penetration depth is $\sim 1/b(\omega)$.

Examples of this kind of medium includes plasmas, and metals such as aluminum, silver and gold which are known to be excellent mirrors for visible and IR radiation.