OPTI 544: Final Exam, May 3, 2019

I

OPTI 544 has explored optical physics in the context of several approximations, descriptions and pictures. In the following provide brief verbal answers to each question.

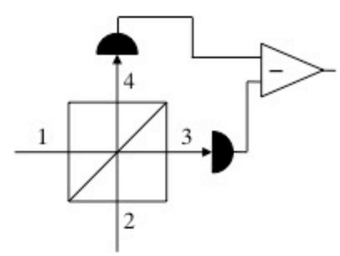
(a) What is meant by fully quantum, semi-classical and fully classical descriptions of light, matter and the interaction between the two. (5%)

(b) Under what conditions is it appropriate to use a semiclassical rather than fully quantum description. Be as specific as possible. (5%)

(c) Under what conditions is it appropriate to use a fully classical rather than semi-classical description. Be as specific as possible. (5%)

(d) A fully quantum and/or semi-classical description of the combined atom-light system can sometimes - but not always - be formulated in terms of a state vector and a Schrödinger equation. What might make such a formulation insufficient? If so, what replaces state vectors and the Schrödinger equation in the formalism? (5%)

II



Consider a homodyne detection setup as shown on the right, consisting of a 50/50 beam splitter and a detector that measures the difference between photon numbers in ports 3 and 4. The input is a product of a coherent state in port 1 (the signal) and a coherent in port 2 (the local oscillator), $|\Psi_{in}\rangle = |\alpha_S\rangle_1 |\alpha_{LO}\rangle_2$.

(a) The detector output is a measurement of the observable $\hat{S} = \hat{N}_3 - \hat{N}_4$. Relying as much as possible on results from past homework, write down (don't derive) expressions for $\langle \hat{S} \rangle$ in terms of α_3, α_4 and in terms of α_S, α_{LO}. (10%)

(b) Relying again on past homework, write down (don't derive) an expression for the measurement variance ΔS^2 in terms of α_S, α_{LO}. (10%)

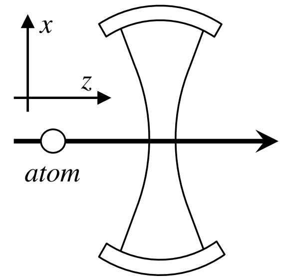
(c) Now let $\alpha_S = |\alpha_S| e^{i\phi_S}$, $\alpha_{LO} = |\alpha_{LO}| e^{i\phi_{LO}}$, where α_{LO} and ϕ_{LO} are known. Find an expression for $\langle \hat{S} \rangle$ in terms of the magnitudes $|\alpha_S|$, $|\alpha_{LO}|$ and phases ϕ_S, ϕ_{LO}. Next, assume $\phi_{LO} = 0$. If the only thing we know about the signal is that $|\alpha_S| \neq 0$, is it possible to determine the unknown phase ϕ_S given $\langle \hat{S} \rangle$? (10%)

A better phase measurement can be achieved by adjusting the (known) phase of the local oscillator such that $\langle \hat{S} \rangle = 0$. At that point we have $\phi_S - \phi_{LO} = 0$ and therefore $\phi_S = \phi_{LO}$. The adjustment is done iteratively while repeating the basic experiment, and the overall strategy is known as an "adaptive measurement".

(d) The resolution of an adaptive measurement is the differential phase $\delta\phi_S = \phi_S - \phi_{LO}$ for which $\langle \hat{S}(\delta\phi_S) \rangle^2 = \Delta S^2$. Find $\delta\phi_S$ as a function of the mean photon number in the signal, in the limit $|\alpha_S| << |\alpha_{LO}|$. (10%)

III

Consider the following scenario in which a two-level atom initially in the ground state travels along the z-axis and crosses the cavity mode parallel to the wave fronts as shown in the figure. For simplicity we assume the cavity field is resonant with the atomic transition, that the transit time is short enough to ignore decay of both the atomic excited state and the intracavity light field, and that light induced forces are small enough for the atomic velocity v to remain constant.



We begin with a classical description of the intracavity field, in which case the atom-light interaction is characterized by a real-valued Rabi frequency $\chi(z) = \chi \exp(-z^2/2\sigma^2)$.

(a) Write down an expression for the angle θ between the Bloch vectors before and after the atom has crossed the cavity, as a function of χ, σ and v. For an atom initially in the ground state, sketch the corresponding trajectory on the Bloch sphere. (10%)

(b) Based on your answer in (a), find the velocities v_g at which the atom will exit the cavity in the ground state. Similarly, find the velocities v_e at which the atom will exit in the excited state. Make a plot that shows the probability P_e for the atom to exit in the excited state as function of v, without worrying too much about the detailed behavior as $v \to 0$. (10%)

Next, we switch to a quantum description of the intracavity field, in which case the atom-light interaction is characterized by a coupling strength $g(z) = g\exp(-z^2/2\sigma^2)$.

(c) For an atom initially in the ground state and the cavity field initially in a number state $|n\rangle$, find the velocities v_g and v_e as function of n. (10%)

(d) Assume the cavity field is known to be in either the $|n=0\rangle$ or $|n=1\rangle$ state. Based on your results in (c), describe how you might determine with complete certainty which of these states is actually present in the experiment, by shooting a single atom though the cavity and measure whether it exits in the ground or excited state. (10%)

Helpful Math: $\int_{-\infty}^{\infty} Exp(-a^2b^2)\,da = \sqrt{\pi/b^2}$