

**OPTI 544: Homework Set #9**  
**Posted April 29, Due May 6.**

**- Keep a copy of your Solution Set -**

**I**

A Cs atom with transition wavelength  $\lambda = 852\text{nm}$  and excited state decay rate  $A_{21} = 3.28 \times 10^7/\text{s}$  is trapped at an antinode of the field in a resonant optical cavity with length  $l = 100\mu\text{m}$  and effective cross sectional area  $A = \pi (2\mu\text{m})^2$ . We assume the field polarization is parallel to the electric dipole matrix element, and that the cavity mirrors are perfect, lossless reflectors.

- (a) Calculate the vacuum Rabi frequency and compare to  $A_{21}$ . Is the system in a regime that permits the observation of coherent vacuum Rabi oscillations?
- (b) A weak probe beam is passed through the cavity. Sketch the transmission as the probe frequency  $\omega$  is tuned around the atom/cavity resonance frequency  $\omega_0$ . What phenomenon are you seeing?

**II**

In the following, consider a normal mode of the electromagnetic field in a cavity that contains *two* identical 2-level atoms. For simplicity we assume the cavity mode is resonant with the atomic transition frequency,  $\omega = \omega_{21}$ , and that the interaction strength  $g$  is the same for both atoms.

- (a) Write down the single-mode, two-atom Jaynes-Cummings Hamiltonian in terms of creation and annihilation operators for the field, and atomic operators  $\hat{\sigma}_{\pm,c}^{(i)}$ , where the index  $i = 1, 2$  refers to the two atoms. (10%)
- (b) The Hamiltonian from (a) has the form  $\hat{H} = \hat{H}_0 + \hat{H}_{AF}$ . Use the notation  $|n, j, k\rangle$  for the eigenstates of  $\hat{H}_0$ , where  $n$  is the photon number, and  $j, k$  are the states of the first and second atom respectively. Find the eigenvalues  $\lambda$  and  $\lambda_q, q = 0, \pm 1$  of  $\hat{H}$  corresponding to coupled atom-cavity states that have zero or one quantum of excitation, respectively. Then draw a level diagram showing the dressed-state energies for the zero- and one-excitation parts of the two-atom Jaynes-Cummings ladder. (9%)
- (c) Sketch the expected transmission of a weak probe beam through the coupled atom-cavity system as the probe frequency is tuned from well below to well above the cavity mode frequency  $\omega$ . No formal calculation is required; rely instead on your physical intuition and your results in (b). (8%)
- (d) Returning to (c), find the eigenstate associated with the eigenvalue  $\lambda_0$  in the single-excitation manifold. Based on your result, how might you revise your sketch from (c)? (8%)