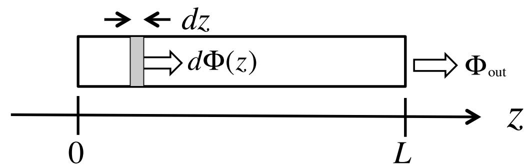
I



Consider the optical amplifier sketched on the left. The 4-level gain medium is unpolarized, $\sigma = \lambda^2/2\pi$, with decay and pump rates $\Gamma_{32} >> \Gamma_{10} >> P >> \Gamma_{21}$. It lies within a waveguide of length $L = 0.05\text{m}$ and cross section $A = 10^{-10}\text{m}^2$. The density of active atoms is $N = 10^{15}\text{m}^{-3}$, the wavelength and frequency of the amplifier transition are $\lambda = 850\text{nm}$ and $\omega_{21} = 2.22\times10^{15}/\text{s}$, the decay $\Gamma_{21} = 3\times10^7/\text{s}$, and $P = 100\times\Gamma_{21}$. Spontaneous photons from the gain medium are captured with an efficiency $\eta = 10^{-3}$ into the guided mode propagating in the $+z$ direction. There is no input, that is, no light coupled into the amplifier from the outside.

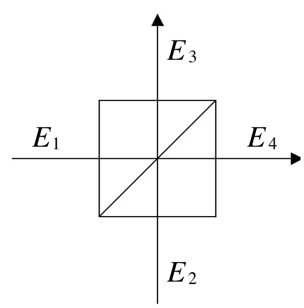
(a) Assuming the total photon flux in the gain medium is far below saturation, find the number density N_2 of atoms in the upper amplifier level, and then the guided wave flux $d\Phi(z) = \Phi dz$ from capture of spontaneously emitted photons within the "slice" of thickness dz at position z. (20%)

(b) Find the small-signal gain g_0 of the amplifier. Assuming the gain medium does not saturate, write down an expression for the output flux $d\Phi_{\text{out}}(z)$ resulting from amplification of the increment $d\Phi(z)$ as it propagates from z to L. (15%)

(c) Finally, integrate $d\Phi_{\text{out}}(z)$ from $z = 0$ to L to find the total flux Φ_{out} at the amplifier output. Then calculate the total power of the amplified spontaneous emission generated the amplifier. Your final answer must be a numerical value with proper units. (15%)

Hint: $\int_0^B e^{A(B-x)}dx = \frac{1}{A}e^{AB}$ for $AB >> 1$

II



Consider in the following a symmetric beamsplitter as shown, with complex transmission and reflection coefficients *t* and *r*.

(a) Given an input state $|\Psi_{in}\rangle = |1\rangle_1|0\rangle_2 = |1,0\rangle$ and general *t* and *r*, write down an expression for the output state $|\Psi_{out}\rangle$ in the basis of two-mode number states $|n_3\rangle|m_4\rangle = |n,m\rangle$. (10%)

(b) Given an input state $|\Psi_{in}\rangle = (|1,0\rangle + |0,1\rangle)/\sqrt{2}$ and the same *t* and *r*, write down an expression for $|\Psi_{out}\rangle$ in the basis $|n_3\rangle|m_4\rangle = |n,m\rangle$. (10%)

(c) If the beamsplitter map is to conserve probability, then it must be a unitary transformation that preserves the normalization of the two-mode states in (a) and (b). Show that this leads to the usual restrictions on *t* and *r*. (15%)

(d) For a 50/50 beamsplitter, you already know what happens when the input state is $|\Psi_{in}\rangle = |1,1\rangle$. Given what you learned in (c), write down the output state $|\Psi_{out}\rangle$ that will result when instead the input state is $|\Psi_{in}\rangle = (|2,0\rangle + |0,2\rangle)/\sqrt{2}$. Explain your result using words and concepts only. Do not resort to any kind of explicit mathematical derivation. (15%)