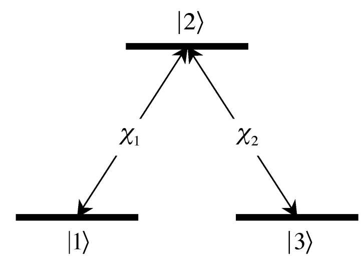
OPTI 544: Homework Set #3
Posted Feb. 19, due Feb. 26.

- Always kleep a copy of your Solution Set -

I

Consider Raman coupling in a three-level atom as indicated in the figure. Both ground-excited state transitions are driven by the same monochromatic plane wave. In contrast to the situation discussed in class, we will assume that both the excited state and Raman detunings are zero, $\Delta = 0$ and $\delta = 0$.



(a) Write out (do not derive) the equations of motion for the probability amplitudes $b_1(t)$, $b_2(t)$ and $b_3(t)$ in the rotating wave approximation.

(b) Let $b_2(0) = 0$. Determine the values of $b_1(0)$ and $b_3(0)$ required to ensure that none of the probability amplitudes change at later times. (The overall complex phase has no physical relevance and can be chosen as you wish.)

Now assume that we prepare an ensemble of three-level atoms in the state found in (b).

(c) What happens to the intensity of the driving field as it propagates through the ensemble?

II

Two separate groups of otherwise identical three-level atoms are prepared in different superposition states. The groups contain equal numbers of atoms.

The first group is prepared in the state $|\psi_1\rangle = \frac{1}{\sqrt{3}}\left(|1\rangle + i\sqrt{2}|2\rangle\right)$.

The second group is prepared in the state $|\psi_2\rangle = \frac{1}{\sqrt{5}}\left((i+1)|1\rangle + |2\rangle - i\sqrt{2}|3\rangle\right)$.

All the atoms are then thoroughly mixed. We will assume that the mixing process does not change the state of the individual atoms in the ensemble.

(a) Find the density matrix for the mixture.

(b) Is the ensemble in a pure or a mixed state? Apply one of the tests for pure states to find out.

III

Consider a two-level atom driven by a monochromatic plane wave, under the conditions used to derive the Rabi solutions in the notes "Atom-Light Interaction: 2-Level Approximation", p. 1-5.

(a) Starting from the basic $\dot{\rho} = -(i/\hbar)[H,\rho]$, show that the components of the density matrix evolve according to the equations

$$\dot{\rho}_{11} = -\frac{i}{2}\left(\chi\rho_{12} - \chi^*\rho_{21}\right), \qquad \dot{\rho}_{22} = \frac{i}{2}\left(\chi\rho_{12} - \chi^*\rho_{21}\right)$$

$$\dot{\rho}_{12} = i\Delta\rho_{12} + \frac{i\chi^*}{2}(\rho_{22} - \rho_{11}), \qquad \dot{\rho}_{21} = -i\Delta\rho_{21} - \frac{i\chi}{2}(\rho_{22} - \rho_{11})$$

(b) Given the constraints on $\hat{\rho}$, how many independent real-valued variables are required to specify the density matrix for a (generally) mixed state. Whan does this tell us about the minimum number of real-valued equations needed to describe the evolution of the system?

(c) The above equations form the basis of much further development, and it is important to think critically about the steps that lead to it. In this spirit, state and discuss the 3 major approximations used to obtain them.

IV

Starting from the equation of motion for the density matrix (including relaxation but setting $\Gamma_1 = \Gamma_2 = 0$ to make things interesting), find the steady state values $\rho_{11}(\infty)$, $\rho_{22}(\infty)$, $\rho_{12}(\infty)$ and $\rho_{21}(\infty)$ for arbitrary resonant Rabi frequency and detuning.