RJ Jones Optical Sciences

OPTI 511L
Fall 2017

Experiment: Saturated Absorption Spectroscopy (2 weeks)

In this experiment we explore the use of a single mode tunable external cavity diode laser (ECDL) to perform saturation spectroscopy in Rb vapor.

Objectives:

I To become familiar with a single frequency external-cavity diode laser (ECDL) and to characterize the laser in terms of frequency tunability.

II To study the level structure of a real atom, as opposed to the familiar, idealized 2-level atom. We will set up and perform spectroscopy in a room temperature, low pressure vapor cell of Rb. This will allow us to observe the Rb Doppler profile, the Lamb dips and cross-over dips. We can also directly observe the effects of power broadening on an atomic transition.

III To understand the complex hyperfine structure of the Rb D_2 transition. This is accomplished using the quantum theory of angular momenta. The insight that we gain is applicable also in other atomic and molecular systems. In particular we will see that the separation between hyperfine states obey Landé's interval rule.

Questions to address in your lab writeup:

(It may be best to work on part of this calculation outside of the lab time.)

(1) Rb atomic transition.

Calculate the Doppler broadened linewidth of a Rb transition in a vapor cell at room temperature ($m_{Rb}=1.42 \times 10^{-25}$ kg). The vapor pressure of Rb in the absorption cell is roughly 4×10^{-10} atmospheres. Calculate the number density of Rb atoms (use ideal gas law). Given a homogeneous linewidth of 6 MHz, and the Doppler width calculated above, what fraction of the atoms interacts with the laser at the center of the Doppler profile? Assuming a resonant scattering cross section of $3\lambda^2/2\pi$, estimate the resonant absorption coefficient for a low intensity beam. What is the attenuation of a resonant beam passing though a 5 cm long Rb cell?

Given the Rb^{87} hyperfine structure shown in fig. 6 (see attached discussion), compute the relative position of the Lamb and cross-over dips in the saturated absorption spectrum, for the multiplets starting from the F=1 and F=2 ground states respectively. Use these numbers to label the positions of the real transitions and cross-over resonances in your saturation absorption experiment.

(2) Make a sketch of the energy level diagrams for Rb^{85}, similar to that shown for Rb^{87} in Fig. 6, that includes the appropriate quantum numbers for the ground and excited hyperfine states F, F' (hint: the nuclear spin of Rb^{85} is I=5/2)

Outline of the experiment:

Calibrate the ECDL laser frequency tuning.

1. The ECDL provides stable single frequency operation. Coarse tuning of the laser (ie its wavelength) can be accomplished by controlling the angle of the external grating. Fine-tuning of the laser frequency is accomplished by changing the laser current or by changing the external cavity length (translating the grating) using a pzt. Using the scanning Fabry-Perot cavity (1.5 GHz or better yet 10 GHz), determine the coefficient that relates the laser frequency shift to changes in the voltage applied to the pzt [Hz/V]. To do this, you may choose to use a function generator to produce a sine wave, and scan the laser frequency across one free-spectral range of the cavity. ***It may be easiest to first scan the Fabry-Perot Interferometer (FPI) in the usual way to confirm the ECDL is running single frequency. Then, shut off the FPI scan, and only scan the laser. Use the FPI to calibrate your scan. Keep this set up so you can make calibrated measurements of the hyperfine splitting's in your spectroscopy experiments.***

Please use caution when operating this laser and ask the TA for instructions on its operation. This is a particularly expensive and sensitive piece of laboratory equipment.

Locate the Doppler-broadened transitions:

2. Use the Agilent OSA to analyze the laser output spectrum. *Take appropriate precautions to avoid unwanted optical feedback to the laser*. Align the laser for single mode operation at the wavelength of the Rb transition by adjusting the laser current and pzt voltage (780.0 nm in air, 780.2 nm in vacuum- be sure to check the OSA for which setting it is in). If needed, ask your TA for assistance in adjusting the external grating. **Make sure the current stays below 120 mA.**

3. Apply a ramping voltage to the PZT controller and slowly scan the laser frequency (at a rate of ~ 10 Hz. Send a weak optical probe beam through the Rb cell and onto a photodiode. The beam intensity should be well below the saturation intensity for the Rb transitions (see attached discussion). Be sure you do not saturate the atomic transition **or** the photodiode so that you obtain a strong absorption signal. Tune the laser frequency and look for absorption using the photodiode measured in transmission. Once you locate the Doppler broadened transitions, maximize the scan *range* such that you can locate the 4 absorption peaks. Depending on how far you can continuously scan the laser without a mode hope, you may or may not be able to see all 4 peaks in a single scan. These 4 peaks correspond to transitions from the two ground state hyperfine levels of Rb^{87} and Rb^{85}. Due to Doppler broadening, you will not be able to resolve the hyperfine structure of the upper levels yet.

   Now, determine which 2 peaks correspond to the 3.0 GHz hyperfine splitting of the Rb^{85} ground state and which belong to the 6.8 GHz hyperfine splitting of the Rb^{87} ground state. Measure the Doppler broadened linewidth for one of these and compare with theory.

Hyperfine structure: saturated absorption spectroscopy

4. **Modify your setup for a saturated absorption experiment** (Fig. 2 gives one simple example for the configuration, your TA will show you other possibilities), taking appropriate care to avoid optical feedback to the laser while still maintaining good beam overlap in the vapor cell. Use the function generator to ramp the laser frequency repeatedly across one of the 4 hyperfine multiplets, and display the transmission vs. frequency on an oscilloscope. You will see a number of dips in the absorption - these lines arise when the frequency of the counter-propagating laser beams coincide with various hyperfine transitions in the Rb vapor. Measure the relative (frequency) separations of these features. Sketch in your lab notebooks what you observe on the scope.

5. ***Identify the transitions you are measuring***. Based on the results from problem (ii) above, assign the observed features to saturation and cross-over lines. Confirm the Landé interval rule by measuring the spacing between transitions (see last page of attached discussion). Are you seeing the multiplet starting from the F=2 or the F=3 ground state of Rb^{85}? Or from the ground state of F=1 or F=2 of Rb^{87}?

Optical pumping:

6. You will observe that the Doppler profiles for the multiplets corresponding to, for example, the F=2 ground state of Rb^{87} is centered on the $F=2 \rightarrow F'=3$ transition (that is, the Doppler broadened background is not symmetric). Can you explain this in terms of ***optical pumping*** between hyperfine states? If not, be sure to discuss with your group and in class (optical pumping is discussed below).

Natural linewidth, power broadening (skip if time is short):

7. If time allows, you may use a variable attenuator to measure the linewidth of one of the saturated absorption features as a function of laser power (if possible). Compare *functional* dependence to theory for the power-broadened 2-level atom. Are you able to extrapolate to zero power and estimate the natural linewidth?

Review of saturated absorption spectroscopy

Please note: in this discussion, Planck's constant is used a number of times. Because of computer and font incompatibilities, $\hbar$ shows up as h.

The Doppler width of a room temperature Rb vapor is greater than the separations between atomic hyperfine levels in the excited state. It is therefore not possible to resolve the excited state hyperfine structure of Rb by measuring the frequency dependent absorption in a Rb vapor. One might speculate if it is possible to reduce the Doppler width of a Rb vapor by cooling the cell. Unfortunately the Doppler width varies slowly with temperature, as $T^{1/2}$, while the vapor pressure varies very fast, approximately as $\exp(-\eta T)$, so the absorption vanishes before the Doppler profile is significantly narrowed. In this experiment we investigate the Rb hyperfine spectrum through the use of a spectroscopic technique known as saturated absorption spectroscopy, which is capable of achieving sub-Doppler resolution.

Case of a 2-level atom. The Lamb dip.

Consider a 2-level atom interacting with two counterpropagating plane waves, in the configuration of fig. 2. We assume that the plane waves both have polarization $\hat{\varepsilon}$ and frequency ω. The waves propagate in the $\pm\hat{z}$ directions, and have amplitudes E_1 and E_2, $|\mathsf{E}_1| >> |\mathsf{E}_2|$. We refer to the plane waves (1) and (2) as the "pump" and the "probe" respectively. The electric fields are:

$$\vec{E}_1 = \hat{\varepsilon}\mathsf{E}_1 e^{-i(\omega t - kz)}, \qquad \vec{E}_2 = \hat{\varepsilon}\mathsf{E}_2 e^{-i(\omega t + kz)}$$

The distribution of atomic velocities along the $\hat{z}$-axis is Maxwell-Boltzmann, i.e. the number density of atoms with velocities in the interval between v and $v + dv$ is

$$N(v)dv = N\sqrt{\frac{m}{2\pi k_B T}}\, e^{-mv^2/2k_B T} dv ,$$

where N is the total number density of atoms. In the presence of the pump wave only, we can find the steady state number densities $N_g(v)$ and $N_e(v)$ of atoms in the ground and excited states (see e. g. "Lasers", Milonni & Eberly, chapter 7):

$$N_e(v) = \frac{N(v)\,\sigma(\omega - kv)\,\Phi}{A + 2\,\sigma(\omega - kv)\,\Phi}, \qquad N_g(v) = N(v) - N_e(v) ,$$

where

$$\sigma(\omega) \;=\; \frac{3\lambda^2}{2\pi}\,\frac{A^2}{4\Delta^2 + A^2}, \qquad \Delta = \omega - \omega_0$$

is the frequency dependent cross section for photon scattering, and $\Phi = I/\hbar\omega$ is the "photon flux". In these expressions ω_0 is the atomic transition frequency, $\lambda = 2\pi c/\omega_0$ is the transition wavelength, and A is the Einstein A-coefficient for spontaneous decay from the upper to the lower state.

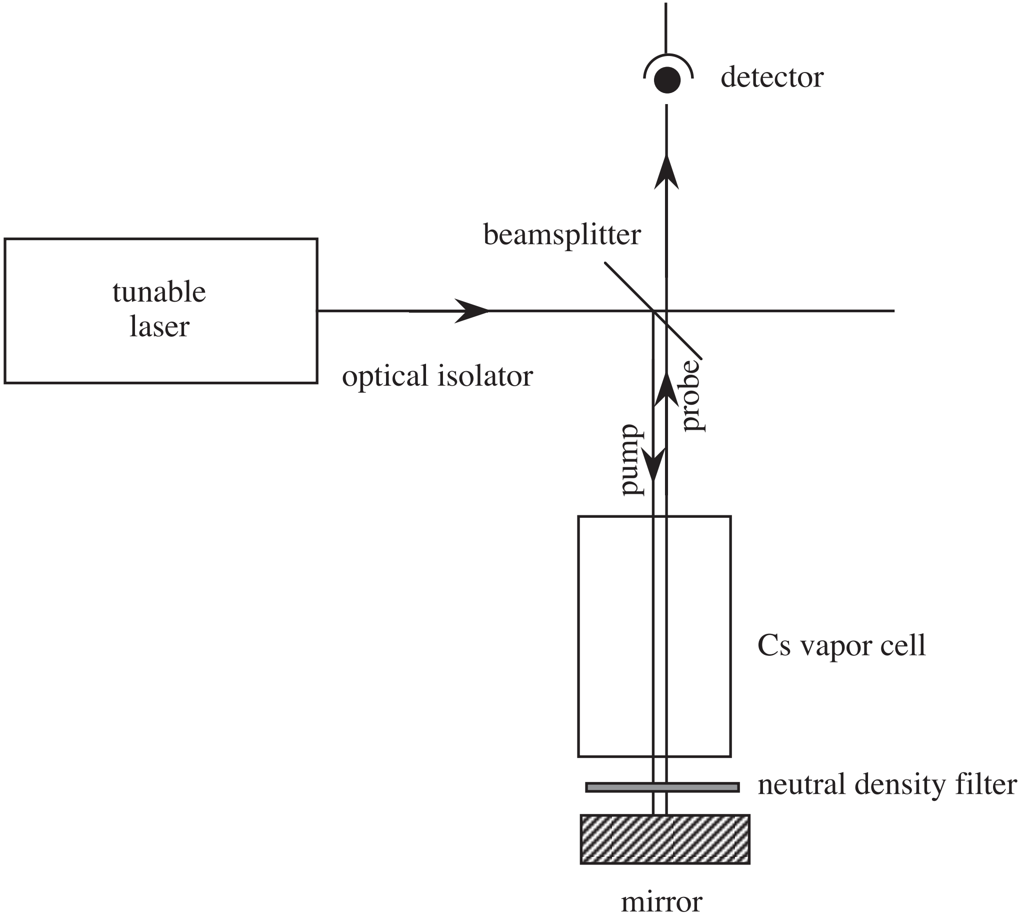

Fig. 2): **One possible saturated absorption setup** (others discussed in class). The Rb vapor in the cell interacts with two counter-propagating laser beams, in the text referred to as the "pump" and the "probe". The frequency-dependent transmitted intensity of the probe is recorded as a function of laser frequency, resulting in a measurement of the saturated absorption spectrum.

Fig. 3 shows $N_g(v)$ for some detuning $\Delta = \omega - \omega_0$ between the pump/probe frequency and the atomic resonance. The phenomenon of "hole-burning" is apparent - there is a sharp depletion of $N_g(v)$ for a velocity class around Δ/k, where atoms are Doppler shifted into resonance with the pump and a substantial fraction become excited. For a probe beam intensity I the FWHM of the hole is

$$\Delta_{FWHM} = A\sqrt{1+\frac{2\sigma(\omega_0)\Phi}{A}} = A\sqrt{1+\frac{I}{I_0}},$$

where $I_0 = A/2\sigma(\omega_0)\hbar\omega$ is the saturation intensity ($\approx 1.1\ \mathrm{mW/cm}^2$ for the Rb D_2 transition). For $I << I_0$ the width of the hole is the natural linewidth A (6 MHz for the Rb D_2 transition).

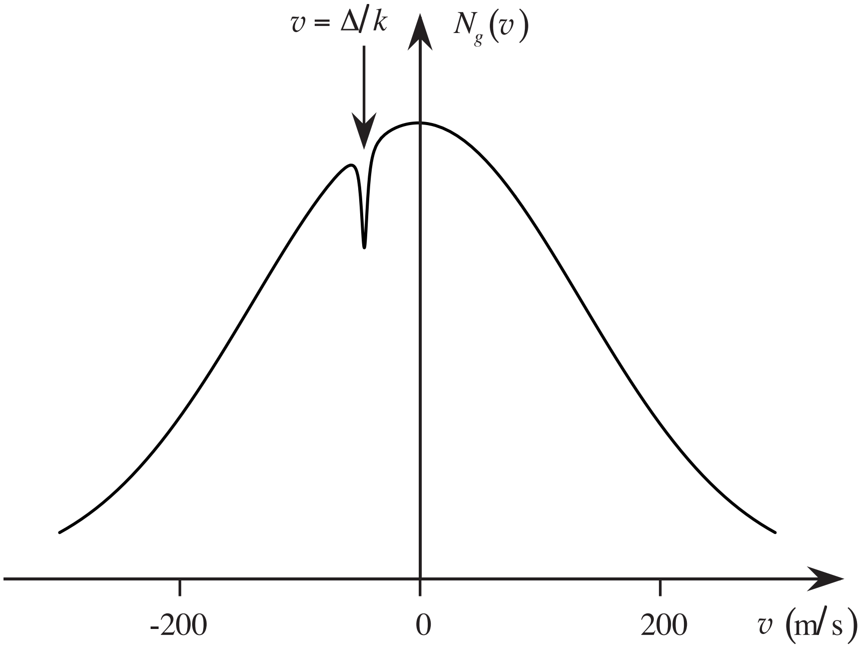

Fig. 3): Hole-burning in $N_g(v)$, the velocity dependent number density of ground state atoms. In this example $T = 300$ K, $\Delta = -10A$ and $I \approx I_0$.

Because the probe beam is much less intense than the pump beam, its presence does not significantly affect the number density of ground and excited state atoms. We then find the following expression for the extinction coefficient experienced by the probe at frequency ω:

$$a(\omega) = \int_{-\infty}^{\infty} \sigma(\omega + kv)\left[N_g(v) - N_e(v)\right] dv$$

The transmission through a cell of length l can now be found as $e^{-a(\omega)l}$. Note that saturated absorption will result in increased transmission, and thus a more intense output beam in the setup of fig. 2.

A more thorough analysis shows that, for $I << I_0$ and in the case of an optically thin gas, there is a minimum in the absorption of the probe with a FWHM equal to the homogeneous linewidth A.

Fig. 4 shows the detuning dependent absorption of the probe in the presence of the pump. A sharp dip - the Lamb dip - is evident for $\Delta = 0$, i. e for a laser frequency close to the atomic transition frequency. We can understand the physical origin of the Lamb dip as follows: the pump depletes $N_g(v)$ for a velocity class around $v_{pump} = \Delta/k$. At the same time the probe interacts with a velocity class around $v_{probe} = -\Delta/k$. As long as these velocity classes are distinct, i. e. as long as the pump and probe interacts with different atoms, the presence of the pump does not affect the absorption of the probe. However, when $\Delta = 0$ we have $v_{pump} = v_{probe}$, i. e. the pump and the probe interacts with the same velocity class. The depletion of ground state atoms caused by the pump then leads to a reduction in the absorption of the probe. By measuring the position of the Lamb dip we can then determine the atomic frequency with a resolution comparable to the natural linewidth A, even if the Doppler broadened linewidth is many times larger.

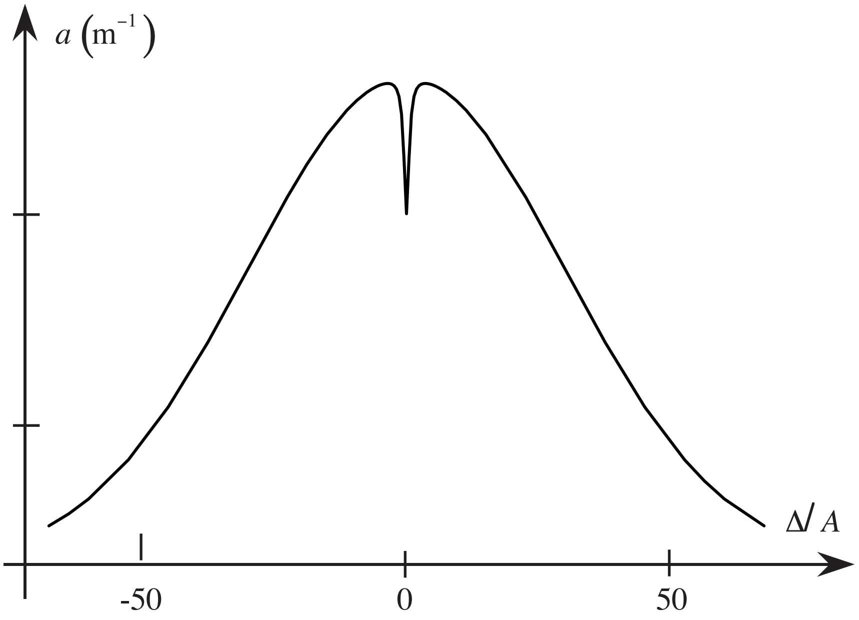

Fig. 4): Lamb dip in a gas of 2-level atoms with the mass and transition parameters of Rb. Parameters as in fig. 3), also $N = 10^{10}$ cm^{-3}.

Case of a 3-level atom. The cross-over dip.

We now consider saturated absorption by a sample of atoms with a single ground state $|g\rangle$ and two excited states $|e1\rangle$, $|e2\rangle$, as shown in fig. 5. As the pump/probe frequency is scanned across the absorption profile one observes two independent Lamb dips, one for $\omega = \omega_1$ and one for $\omega = \omega_2$. As explained above these Lamb dips occur whenever the pump/probe frequency is equal to an atomic transition frequency.

An additional feature is found in the saturated absorption spectrum of this multilevel atom. Consider a velocity class around a velocity v such that $\omega + kv = \omega_2$ and $\omega - kv = \omega_1$. For these atoms the pump will excite the transition $|g\rangle \leftrightarrow |e1\rangle$, and deplete the number density of ground state atoms. This depletion of ground state atoms will in turn lead to a decrease in the absorption of the probe, since fewer atoms are available for excitation on the transition $|g\rangle \leftrightarrow |e2\rangle$ (fig. 5b). Similarly there will be a velocity class around velocity $-v$ where the pump excites the $|g\rangle \leftrightarrow |e2\rangle$ transition and depletes the number density of ground state atoms available for absorption on the $|g\rangle \leftrightarrow |e1\rangle$ (fig. 5c). The result is the so-called cross-over dip in the absorption, which occurs at a pump/probe frequency of $\omega = (\omega_1 + \omega_2)/2$.

More generally, if the atoms in the sample have one ground state and n excited states, there will be n Lamb dips and $n(n-1)/2$ cross-over dips corresponding to all possible ways the upper states can be combined in pairs. For the Rb^{85} D_2 multiplets we have 3 excited states that are allowed by the selection rule $\Delta F = 0, \pm 1$, and therefore we expect 3 Lamb dips and 3 cross-over dips in the absorption profile. Their relative positions can be found based on the separations given in fig. 6.

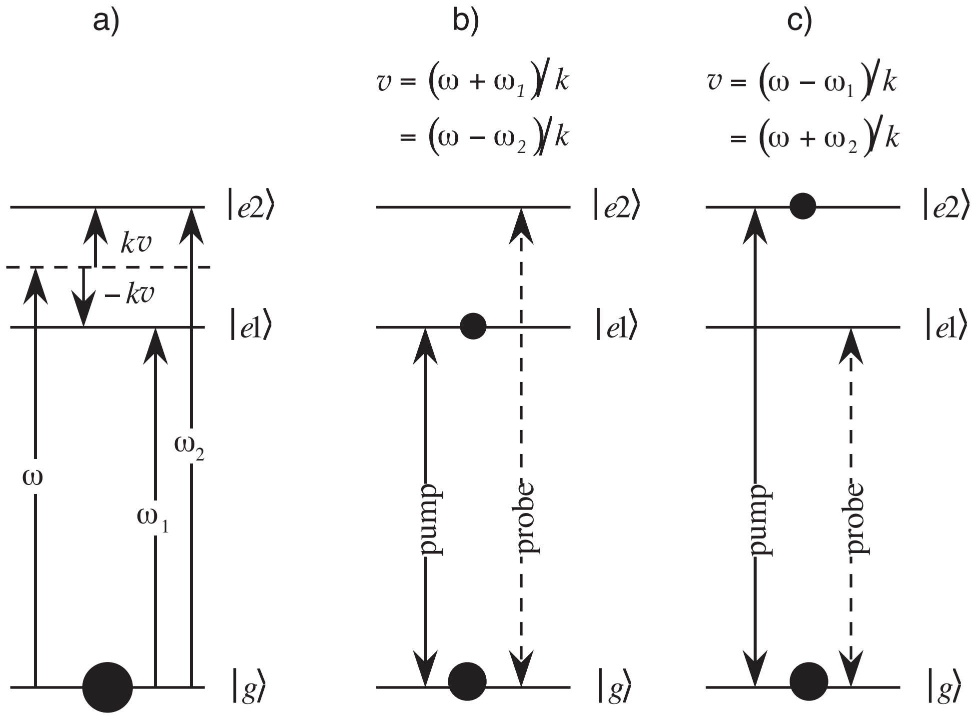

Fig 5): a) 3-level atomic system for which both Lamb and cross-over dips are seen. The cross-over dip occurs when the pump-probe beams at frequency ω are Doppler shifted into resonance with the $|g\rangle \leftrightarrow |e1\rangle$ and $|g\rangle \leftrightarrow |e2\rangle$ transitions. This happens for two velocity classes around $v = (\omega + \omega_1)/k = (\omega - \omega_2)/k$ (b) and $v = (\omega - \omega_1)/k = (\omega + \omega_2)/k$ (c).

Optical Pumping.

Consider a sample of Rb^{87} atoms (see level scheme in fig. 6) initially in the F=2 ground state. If these atoms are excited on the F=2 → F'=3 transition, there will be some population transferred to the F'=3 excited state. Because of the selection rule $\Delta F = 0, \pm 1$, atoms in the F'=3 excited state can only decay to the F=2 ground state (and not the F=1 ground state). Such a transition is referred to as a ***closed transition***, since the total number of atoms in the F=2 and F'=3 states is obviously conserved.

Consider now a sample of Rb atoms initially in the F=2 ground state, but let these atoms be excited to the F'=2 excited state by a laser at the appropriate frequency. Atoms in the F'=2 excited state can decay both to the F=2 and F=1 ground states. An atom that decays to the F=2 ground state can be excited again; however an atom that decays to the F=1 ground state stops interacting with the light field, since all transitions beginning from the F=2 ground state are very far detuned from resonance (see fig. 6). Atoms that decay to the F=1 ground state will therefore not be excited again, and can never be transferred back to the F=2 ground state. Hence, after a few absorption/spontaneous decay cycles, all atoms in the sample will eventually be transferred to the F=1 ground state, and the sample will become transparent. This process is referred to as ***optical pumping***.

Rb atomic structure:

Electron configuration:

Rb has 2 naturally occurring isotopes; Rb^{85} (72.2% abundance, I=5/2) and Rb^{87} (27.8% abundance, I=3/2). The ground state electronic configuration is identical to the Kr atom with an additional 5s electron (i. e. the atom has a single 5s electron outside a core composed of completely filled subshells). The first excited state of the Rb atom corresponds to a transfer of the 5s electron to a 5p state. It has long been known that when one looks at the 5s-5p transition in the spectrum of light emitted from a Rb discharge, one sees two closely spaced resonance lines, historically named the D_1 and D_2 lines. This splitting of the 5s-5p spectral line is known as fine structure. As demonstrated in this experiment, saturated absorption spectroscopy reveals many more closely spaced resonance lines - this splitting in turn is known as hyperfine structure. The hyperfine levels involved in the D_2 transition are shown in fig. 6). All the splittings between the hyperfine levels have very simple physical interpretations in terms of the interaction between the magnetic moments associated with the electron orbital and spin angular momentum, and the angular momentum of the atomic nucleus. To permit such an interpretation, we briefly review the basic elements of angular momentum theory in quantum mechanics.

Orbital angular momentum, the quantum number L:

In quantum mechanics electrons moving in the Coulomb potential created by the nucleus have orbital angular momentum $\hat{\mathbf{L}}$. $\hat{\mathbf{L}}$ is defined as in classical mechanics: $\hat{\mathbf{L}} = \hat{\mathbf{r}} \times \hat{\mathbf{p}}$, but $\hat{\mathbf{L}}$ and the position $\hat{\mathbf{r}}$ and the momentum $\hat{\mathbf{p}}$ are operators. Consider now the component $\hat{L}_z = \hat{y}\hat{p}_x - \hat{x}\hat{p}_y$. The Heisenberg uncertainty relations prevent a simultaneous measurement of e. g. $\hat{x}$ and $\hat{p}_x$; hence one cannot simultaneously measure all three components of $\hat{\mathbf{L}}$. More specifically, one can use the commutation relations for $\hat{\mathbf{r}}$ and $\hat{\mathbf{p}}$ to derive the commutation relations for the components of $\hat{\mathbf{L}}$; this proves that the components of $\hat{\mathbf{L}}$ do not commute and therefore are not simultaneously observable. In other words: an atomic state cannot correspond to well defined values of all three components $\hat{L}_x$, $\hat{L}_y$ and $\hat{L}_z$. If we define $\hat{L}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2$, it can however be shown that atomic states correspond to well defined values of $\hat{L}^2$ and $\hat{L}_z$. This simply means that only the magnitude of $\hat{\mathbf{L}}$ and its projection on *one* axis can be measured simultaneously. The eigenvalues (possible outcomes of a measurement) are

$$\begin{aligned} &\hat{L}^2 : \hbar^2 L(L+1) && L \geq 0,\ integer \\ &\hat{L}_z : \hbar m_L && -L \leq m_L \leq L,\ integer \end{aligned} \qquad (1)$$

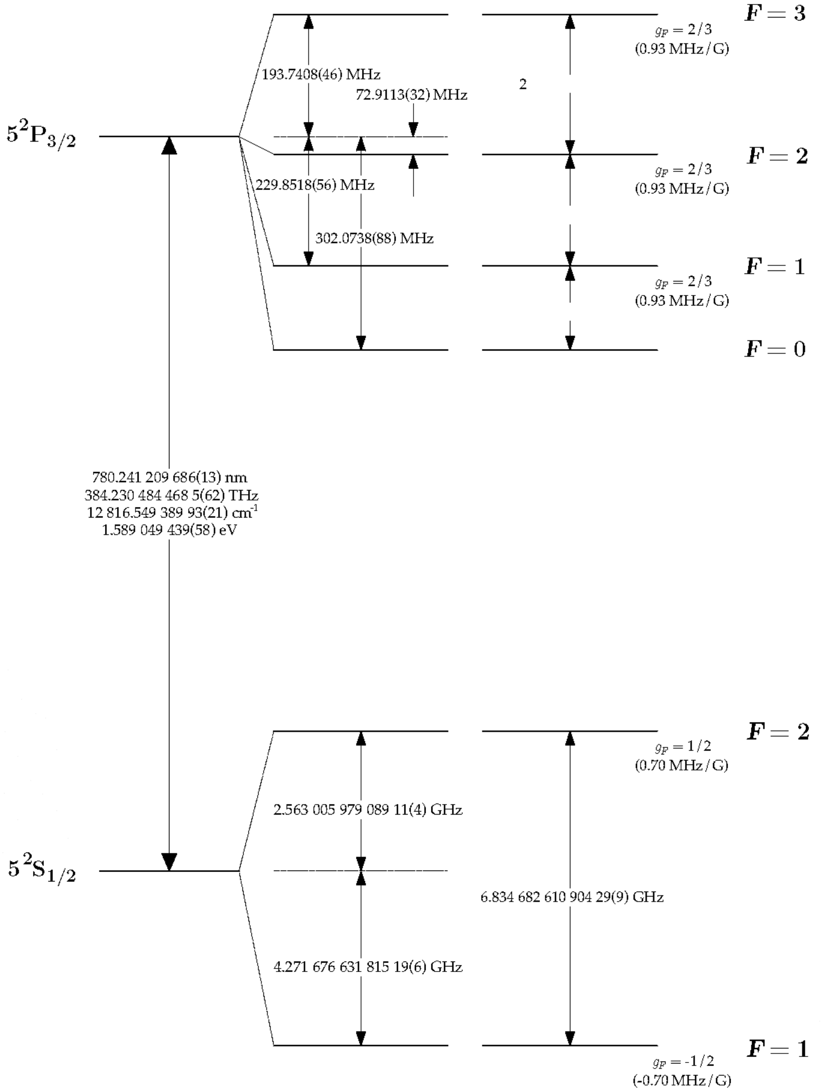

Fig. 6): Rb^{87} hyperfine structure in the ground and excited states. Transitions between these hyperfine states form the D_2 multiplet of resonance lines.

States with $L = 0,1,2,3$ are known as s,p,d,f states respectively. A group of $2L+1$ states corresponding to a particular value of L and the radial quantum number, but different quantum numbers m_L, are referred to as a subshell.

Spin angular momentum, the quantum number S:

Particles such as electrons, protons and neutrons all have an intrinsic angular momentum $\hat{\mathbf{S}}$, the spin. Spin angular momenta behave just like orbital angular momenta, except that spin angular momenta are half-integer, i. e. the eigenvalues of the operators $\hat{S}^2$ and $\hat{S}_z$ are

$$\begin{aligned} &\hat{S}^2 : \hbar^2 S(S+1) = 3\hbar^2/4 && S = 1/2 \\ &\hat{S}_z : \hbar m_S && m_S = \pm 1/2 . \end{aligned} \tag{2}$$

Total electron angular momentum, the quantum number J:

Orbital and spin angular momenta of all the electrons in an atom can be added, resulting in a total angular momentum $\hat{\mathbf{J}} = \hat{\mathbf{L}} + \hat{\mathbf{S}}$. Again we have eigenvalues and quantum numbers

$$\begin{aligned} &\hat{J}^2 : \hbar^2 J(J+1) && |L-S| \le J \le L+S, \ \textit{integer or half - integer} \\ &\hat{J}_z : \hbar m_J && -J \le m_J \le J, \ \textit{integer or half} - \textit{integer} \end{aligned} \tag{3}$$

Whenever two angular momenta $\hat{\mathbf{J}}_1$ and $\hat{\mathbf{J}}_2$ are added to yield a total angular momentum $\hat{\mathbf{J}} = \hat{\mathbf{J}}_1 + \hat{\mathbf{J}}_2$, then the quantum number J takes on all the values $|J_1 - J_2| \le J \le J_1 + J_2$. If for example $J_1 = 4,\ J_2 = 2$, then the possible values of J are $2,3,4,5,6$.

We can now add the angular momenta of all the electrons in an atom. There is some significance to the order in which this is done; we will however ignore this question here. One important result is that the total angular momentum of all the electrons in a filled subshell is zero. This is easy to show by arranging electrons into pairs with quantum numbers $(m_L, -m_L)$ and $(m_S, -m_S)$; hence $\sum m_L + \sum m_S = 0$ and $\hat{J}_z = 0$. But if the only allowed eigenvalue of $\hat{J}_z$ is zero, then $J = 0$. For an atom such as Rb, consisting of a series of closed shells plus a single unpaired electron with orbital and spin angular quantum numbers L and $S = 1/2$, the possible values of the quantum number for the total angular momentum are $J = |L \pm 1/2|$.

Fine-structure:

With an electron in an atomic state one can associate a probability current J that circulates around the atomic nucleus - this is the quantum mechanical analogy of the classical concept of an orbiting particle. Since an electron is charged, the probability current is equivalent to a circulating electrical current eJ. This current in turn gives rise to a magnetic dipole moment $\boldsymbol{\mu}_L \propto \hat{\mathbf{L}}$.

Similarly, an electron has an intrinsic magnetic moment $\boldsymbol{\mu}_S = \mu_B \hat{\mathbf{S}}$, where μ_B is the Bohr magneton.

Depending on the relative orientation of the angular momenta $\hat{\mathbf{L}}$ and $\hat{\mathbf{S}}$, there is an interaction energy associated with the magnetic dipoles $\boldsymbol{\mu}_L$ and $\boldsymbol{\mu}_S$. Thus there is a shift ΔE of the energy of the atomic states which is proportional to $\boldsymbol{\mu}_L \cdot \boldsymbol{\mu}_S$, and therefore proportional to $\hat{\mathbf{L}} \cdot \hat{\mathbf{S}}$. But if we remember that

$$\hat{\mathbf{L}} \cdot \hat{\mathbf{S}} = \frac{\hat{J}^2 - \hat{L}^2 - \hat{S}^2}{2}, \tag{4}$$

and also remember that the eigenvalues of the operators $\hat{J}^2$, $\hat{L}^2$ and $\hat{S}^2$ are given by eqs. 1, 2 and 3, then we see that

$$\Delta E \propto J(J+1) - L(L+1) - 3/4. \tag{5}$$

This allows us to compute the relative values of the fine-structure shift for states with the same quantum number L but different values of the quantum number J.

Consider now the ground state of the Cs atom. Because the unpaired electron is in a $6s$ state we have $L = 0$, $S = 1/2$ and $J = 1/2$. Because there is only one possible value of J there is only one ground state. Also, the value of the energy shift is $\Delta E = 0$ (Note: there are other contributions to the fine-structure in atoms that do lead to a shift of the ground state; these contributions do not produce a splitting of states and are therefore ignored here).

Consider then the first excited state of the Rb atom. The unpaired electron is in the 5p state and we have $L = 1$, $S = 1/2$ and $J = 1/2, 3/2$. The two states corresponding to different values of J are shifted by $\Delta E_{1/2} = -2\kappa_{6p}$ and $\Delta E_{3/2} = \kappa_{6p}$, where κ_{6p} is a constant that depends in a very complicated way on the electronic structure of the Rb atom. For atomic species (such as Rb) with a valence subshell that is less than half filled, the constant κ is positive, whereas for a valence subshell that is more than half filled κ is negative. We now see why the Rb 5s – 5p transition is actually split into two lines with a separation given by the upper state fine-structure splitting.

From eq. 5 we see that the energy splitting of two states with angular momentum quantum numbers J and $J - 1$ is

$$\delta E_J = \Delta E_J - \Delta E_{J-1} \propto J(J+1) - (J-1)J \propto J, \tag{6}$$

i. e. the fine-structure splitting is proportional to the total angular momentum of the upper state. This result is known as Landé's interval rule. For Rb the fine structure splitting of the 5s – 5p transition is approximately 15 nm. We are therefore not able to observe fine structure in this experiment due to the limited tuning range of the laser.

Hyperfine structure:

We have so far determined the (relative) fine structure splitting of atomic states characterized by different quantum numbers J for the total electron angular momentum. We must now take into account the interaction between the total magnetic moment of the electrons and the nuclear magnetic moment. The nuclear magnetic moment is proportional to the nuclear spin $\hat{\mathbf{I}}$, and the resulting interaction energy is $\Delta E_F \propto \hat{\mathbf{J}} \cdot \hat{\mathbf{I}}$. For Rb^{87} the nuclear spin quantum number is I=3/2.

Total atomic angular momentum, the quantum number F:

As was the case for fine-structure, we obtain hyperfine states with angular momentum $\hat{\mathbf{F}} = \hat{\mathbf{I}} + \hat{\mathbf{J}}$, where

$$\begin{array}{lll} \hat{F}^2: & \hbar^2 F(F+1) & |I-J| \le F \le I+J, \textit{ integer} \\ \hat{F}_z: \hbar m_F & & -F \le m_F \le F, \textit{ integer} \end{array} \qquad (7)$$

(integer values occur because both I and J are half-integers).

Hyperfine structure:

We know that the Rb^{87} ground state has $J = 1/2$ and therefore two possible values F=1,2 of the total angular momentum. The ground state thus is split into two hyperfine states. The $J' = 3/2$ excited state is split into four states with F'=0,1,2,3. This is the hyperfine structure shown in fig. 6. Proceeding as for the fine structure splitting, we see that the hyperfine splitting between states F and $F-1$ also obeys The Landé interval rule

$$\delta E_F = \Delta E_F - \Delta E_{F-1} \propto F(F+1) - (F-1)F \propto F \,. \qquad (8)$$

Thus the splittings of the four excited state hyperfine levels have relative magnitudes 3:2:1 (see fig. 6). In our experiment we observe transitions between levels in the ground and the $J' = 3/2$ excited state hyperfine manifolds; the number and relative separation of saturated absorption features in our spectrum can therefore be explained simply in terms of hyperfine states and Landé's interval rule.