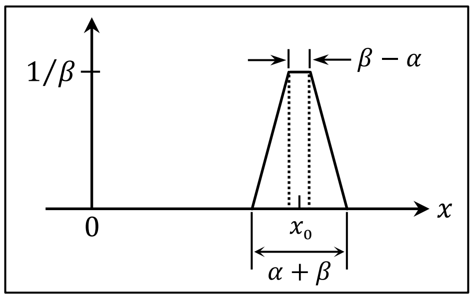
Problem 2) Either of the rectangular functions may be selected to flip around the vertical axis, then shift along the x-axis. It is easier to flip $\alpha^{-1}\text{rect}(x/\alpha)$ as it does *not* change upon flipping. The two functions remain fully separated so long as the shift x is either less than $x_0 - ½(\alpha + \beta)$ or greater than $x_0 + ½(\alpha + \beta)$. In between these two shifts, the functions overlap, either fully or partially. Multiplication then yields the constant product $(\alpha\beta)^{-1}$ of the magnitudes within the overlap region, and zero outside that region. The integrated product of the overlapping rectangles then rises linearly between $x = x_0 - ½(\alpha + \beta)$ and $x = x_0 - ½(\alpha + \beta) + \alpha$. Afterward, the overlap remains complete, up until $x = x_0 + ½(\alpha + \beta) - \alpha$. Subsequently, the overlap integral declines linearly, reaching zero at $x = x_0 + ½(\alpha + \beta)$. A plot of the integrated product of the two functions versus the shift x is shown below.



Note that the area of the trapezoid equals 1. When $\alpha \to 0$ and $\beta \to 0$, the trapezoid, always symmetric around its center at $x = x_0$, becomes tall and narrow while maintaining its area at 1. Thus, the convolution of the original rectangular pulses converges to $\delta(x - x_0)$.