Problem 1) Recalling that

$$\partial z/\partial x = \partial(x + \mathrm{i}y)/\partial x = 1, \tag{1}$$

$$\partial z^*/\partial x = \partial(x - \mathrm{i}y)/\partial x = 1, \tag{2}$$

$$\partial z/\partial y = \partial(x + \mathrm{i}y)/\partial y = \mathrm{i}, \tag{3}$$

$$\partial z^*/\partial y = \partial(x - \mathrm{i}y)/\partial y = -\mathrm{i}, \tag{4}$$

we write

$$\partial f/\partial x = (\partial f/\partial z)(\partial z/\partial x) + (\partial f/\partial z^*)(\partial z^*/\partial x) = \partial f/\partial z + \partial f/\partial z^*, \tag{5}$$

$$\partial f/\partial y = (\partial f/\partial z)(\partial z/\partial y) + (\partial f/\partial z^*)(\partial z^*/\partial y) = \mathrm{i}(\partial f/\partial z - \partial f/\partial z^*). \tag{6}$$

i) $f(z) = |z|^2 = x^2 + y^2$. Equations (5) and (6) yield

$$\partial f/\partial x = \partial(zz^*)/\partial x = z^* + z = 2x, \tag{7}$$

$$\partial f/\partial y = \partial(zz^*)/\partial y = \mathrm{i}(z^* - z) = 2\mathrm{y}. \tag{8}$$

These agree with the results of direct calculation, $\partial(x^2 + y^2)/\partial x = 2x$ and $\partial(x^2 + y^2)/\partial y = 2y$.

ii) $f(z) = |z|^4 = x^4 + y^4 + 2x^2y^2$. Equations (5) and (6) yield

$$\partial f/\partial x = \partial(z^2z^{*2})/\partial x = 2zz^{*2} + 2z^2z^* = 2zz^*(z^* + z) = 4|z|^2\mathrm{Re}(z) = 4x(x^2 + y^2), \tag{9}$$

$$\partial f/\partial y = \partial(z^2z^{*2})/\partial y = \mathrm{i}(2zz^{*2} - 2z^2z^*) = \mathrm{i}2zz^*(z^* - z) = 4|z|^2\mathrm{Im}(z) = 4y(x^2 + y^2). \tag{10}$$

Direct calculations agree, namely, $\partial f/\partial x = 4x^3 + 4xy^2$ and $\partial f/\partial y = 4y^3 + 4x^2y$.

iii) $f(z) = e^{|z|^2+z} = e^{x^2+y^2+x+\mathrm{i}y} = e^{x^2+y^2+x}(\cos y + \mathrm{i}\sin y)$. Equations (5) and (6) yield

$$\partial(e^{zz^*+z})/\partial x = (z^* + 1)e^{zz^*+z} + ze^{zz^*+z} = (z + z^* + 1)e^{zz^*+z} = (2x + 1)e^{x^2+y^2+x+\mathrm{i}y}. \tag{11}$$

$$\begin{aligned}\partial(e^{zz^*+z})/\partial y &= \mathrm{i}\left[(z^* + 1)e^{zz^*+z} - ze^{zz^*+z}\right] = \mathrm{i}(z^* - z + 1)e^{zz^*+z} = (2y + \mathrm{i})e^{x^2+y^2+x+\mathrm{i}y}\\ &= [2y(\cos y + \mathrm{i}\sin y) + (\mathrm{i}\cos y - \sin y)]e^{x^2+y^2+x}\\ &= [(2y\cos y - \sin y) + \mathrm{i}(2y\sin y + \cos y)]e^{x^2+y^2+x}.\end{aligned} \tag{12}$$

Direct calculations agree, namely,

$$\partial f/\partial x = (2x + 1)e^{x^2+y^2+x}(\cos y + \mathrm{i}\sin y). \tag{13}$$

$$\partial f/\partial y = 2ye^{x^2+y^2+x}(\cos y + \mathrm{i}\sin y) - e^{x^2+y^2+x}(\sin y - \mathrm{i}\cos y). \tag{14}$$