Problem 2) a) Separating the terms of the binomial expansion that have even k from those with odd k, we arrive at

$$(x+y)^{2n} = \sum_{k=0}^{2n} \binom{2n}{k} x^k y^{2n-k} = \sum_{k=0}^{n} \binom{2n}{2k} x^{2k} y^{2(n-k)} + \sum_{k=0}^{n-1} \binom{2n}{2k+1} x^{2k+1} y^{2(n-k)-1}.$$

b) If $x = 1$ and $y = -1$, then the first sum on the right-hand side, whose powers of y are even, will be a positive number, whereas the second sum, having odd powers of y, will be negative. Since the sum of these two terms is now going to be zero, their magnitudes must be equal.

c) If $x = y = 1$, the odd and even sums on the right-hand side of the above equation will be positive and add up to 2^{2n}. Since these two terms are equal according to part (b), each must amount to one-half of 2^{2n}, which is 2^{2n-1}.

d) Separating the terms of the expansion with even k from those with odd k, we arrive at

$$(x+y)^{2n+1} = \sum_{k=0}^{2n+1} \binom{2n+1}{k} x^k y^{2n+1-k} = \sum_{k=0}^{n} \binom{2n+1}{2k} x^{2k} y^{2(n-k)+1} + \sum_{k=0}^{n} \binom{2n+1}{2k+1} x^{2k+1} y^{2(n-k)}.$$

When $x = 1$ and $y = -1$, the first sum on the right-hand side of the above equation, whose powers of y are odd, will be a negative number, whereas the second sum, having even powers of y, will be positive. Since the sum of these two terms is now going to be zero, their magnitudes must be equal. In contrast, when $x = y = 1$, the odd and even sums on the right-hand side of the equation will be positive and add up to 2^{2n+1}. Each of the two sums thus amounts to one-half of 2^{2n+1}, which is 2^{2n}.