Problem 1) a) For $n=0$, we have $2^{n+2}+3^{2n+1}=2^2+3=7$, which is divisible by 7. Similarly, for $n=1$, we have $2^{n+2}+3^{2n+1}=2^3+3^3=35$, which is also divisible by 7. Suppose now that the statement is true for n. Then, for $n+1$ we will have

$$2^{(n+1)+2}+3^{2(n+1)+1}=2(2^{n+2})+9(3^{2n+1})=2(2^{n+2}+3^{2n+1})+7(3^{2n+1}).$$

The above expression is divisible by 7, simply because its first term is a multiple of 7 by the induction hypothesis, while its second term is clearly a multiple of 7. The proof is thus complete.

b) For $n=0$, we have $|\sin(nx)|=|\sin(0)|=0$ and $n|\sin(x)|=0$; therefore, the statement is true. Similarly, for $n=1$, we have $|\sin(nx)|=|\sin(x)|$, which equals $n|\sin(x)|=|\sin(x)|$. We now assume the validity of the statement for $n>1$, and proceed to confirm it for $n+1$. In the process, we invoke the well-known identities $|a+b|\le|a|+|b|$ and $|\cos(nx)|\le 1$, as follows:

$$|\sin[(n+1)x]|=|\sin(nx)\cos(x)+\cos(nx)\sin(x)|\le|\sin(nx)\cos(x)|+|\cos(nx)\sin(x)|$$
$$\le|\sin(nx)|+|\sin(x)|\le n|\sin(x)|+|\sin(x)|=(n+1)|\sin(x)|.$$

The proof is now complete.