Problem 1) a) $\int_{-L/2}^{L/2} e^{\mathrm{i}2\pi sx}\mathrm{d}x = (\mathrm{i}2\pi s)^{-1} e^{\mathrm{i}2\pi sx}\Big|_{x=-L/2}^{L/2} = (e^{\mathrm{i}\pi sL} - e^{-\mathrm{i}\pi sL})/(\mathrm{i}2\pi s) = \frac{\sin(\pi Ls)}{\pi S}$.

The standard sinc function is defined as $\mathrm{sinc}(s) = \sin(\pi s)/(\pi s)$, which is an even function of s, whose area equals 1, and whose value at $s = 0$ also equals 1. The above integral may thus be written as $L\,\mathrm{sinc}(Ls)$, which, in the limit when $L \to \infty$, becomes a tall, narrow, even function of s, whose area $\int_{-\infty}^{\infty} L\,\mathrm{sinc}(Ls)\mathrm{d}s$, being the same as the area under $\mathrm{sinc}(s)$, equals 1. (The last assertion is readily proven by changing the integration variable to $s' = Ls$.) Consequently,

$$\int_{-\infty}^{\infty} e^{\mathrm{i}2\pi sx}\mathrm{d}x = lim_{L\to\infty} \int_{-L/2}^{L/2} e^{\mathrm{i}2\pi sx}\mathrm{d}x = lim_{L\to\infty}\{L\,\mathrm{sinc}(Ls)\} = \delta(s).$$

b) $$H(s) = \mathcal{F}\{f(x)g(x)\} = \int_{-\infty}^{\infty} f(x)g(x)e^{-\mathrm{i}2\pi sx}\mathrm{d}x$$
$$= \int_{-\infty}^{\infty}\Big[\int_{-\infty}^{\infty} F(s')e^{\mathrm{i}2\pi s'x}\mathrm{d}s'\Big]\Big[\int_{-\infty}^{\infty} G(s'')e^{\mathrm{i}2\pi s''x}\mathrm{d}s''\Big]e^{-\mathrm{i}2\pi sx}\mathrm{d}x$$
$$= \iiint_{-\infty}^{\infty} F(s')G(s'')e^{\mathrm{i}2\pi(s'+s''-s)x}\,\mathrm{d}s'\mathrm{d}s''\mathrm{d}x.$$

c) $$H(s) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(s')G(s'')\Big[\int_{-\infty}^{\infty} e^{\mathrm{i}2\pi(s''+s'-s)x}\mathrm{d}x\Big]\mathrm{d}s'\mathrm{d}s''$$ ← substitute the result of part (a)
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(s')G(s'')\delta(s''+s'-s)\mathrm{d}s'\mathrm{d}s''.$$

d) $$H(s) = \int_{-\infty}^{\infty} F(s')\Big\{\int_{-\infty}^{\infty} G(s'')\delta[s''-(s-s')]\mathrm{d}s''\Big\}\mathrm{d}s'$$ ← invoke the sifting property of δ-function
$$= \int_{-\infty}^{\infty} F(s')G(s-s')\mathrm{d}s' = F(s) * G(s).$$