Problem 2) To find the area of the ellipse, we first integrate along the y-axis at an arbitrary location on the x-axis, then integrate along x from $x = -a$ to a. The domain of the integral along y is obtained by solving the equation of the ellipse for $y(x)$; that is, $y = \pm b\sqrt{1-(x/a)^2}$. We will have

$$A = \iint_{\text{ellipse}'\text{s surface}} \mathrm{d}x\mathrm{d}y = \int_{x=-a}^{a} \int_{y=-b\sqrt{1-(x/a)^2}}^{b\sqrt{1-(x/a)^2}} \mathrm{d}x\mathrm{d}y = \int_{x=-a}^{a} 2b\sqrt{1-(x/a)^2}\mathrm{d}x$$

$$= 2b\int_{\theta=-\pi/2}^{\pi/2} \sqrt{1-\sin^2\theta}\,(a\cos\theta)\mathrm{d}\theta = 2ab\int_{\theta=-\pi/2}^{\pi/2} \cos^2\theta\,\mathrm{d}\theta$$

$$= ab\int_{\theta=-\pi/2}^{\pi/2} (1+\cos 2\theta)\mathrm{d}\theta = ab\left[\pi + ½\sin(2\theta)\Big|_{\theta=-\pi/2}^{\pi/2}\right]$$

$$= \pi ab + ½ab[\sin(\pi) - \sin(-\pi)] = \pi ab + ab\,\cancelto{0}{\sin\pi} = \pi ab.$$