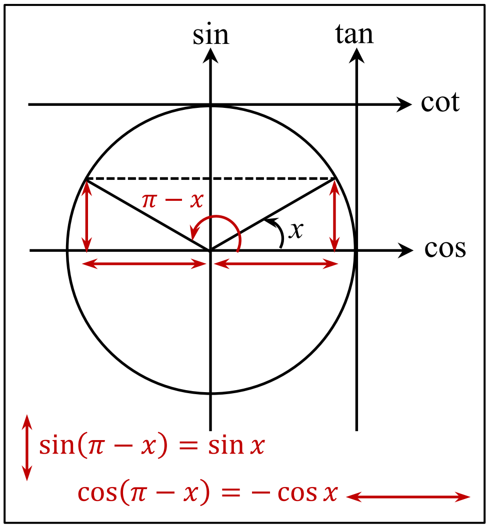
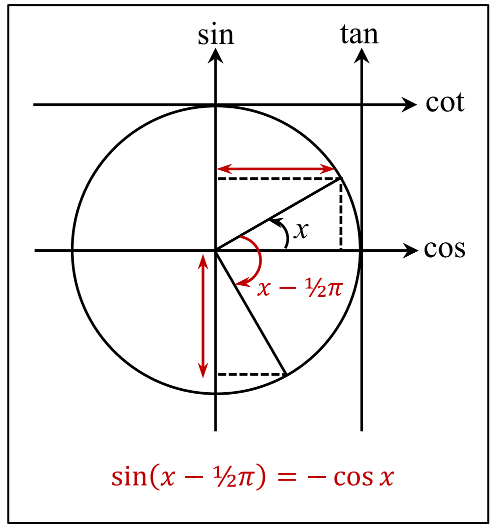
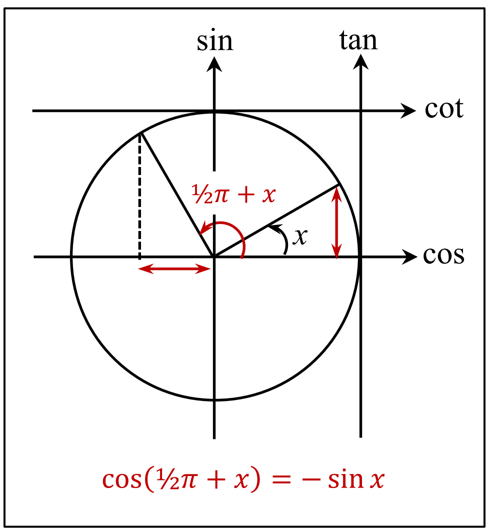
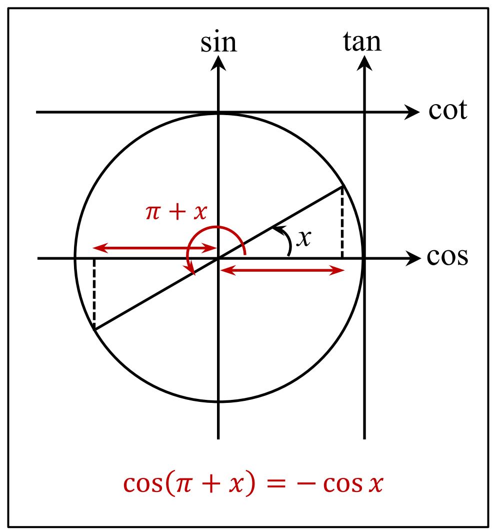
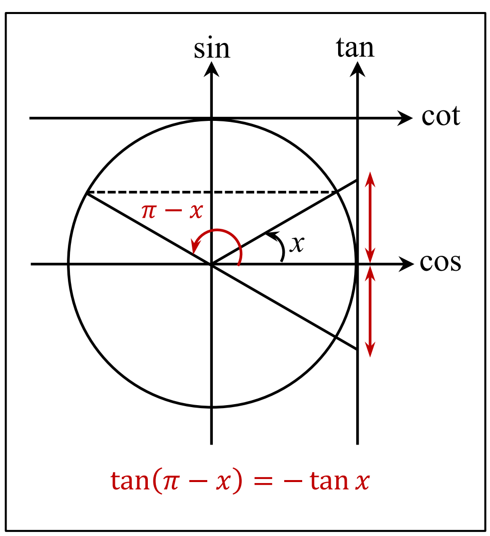
Problem 1) a) In the diagrams below, the red arrows mark the quantities of interest in each case.











b) $e^{\mathrm{i}(x\pm y)} = e^{\mathrm{i}x}e^{\pm\mathrm{i}y} \quad\rightarrow\quad \cos(x\pm y) + \mathrm{i}\sin(x\pm y) = (\cos x + \mathrm{i}\sin x)(\cos y \pm \mathrm{i}\sin y)$

$\rightarrow\quad \cos(x\pm y) + \mathrm{i}\sin(x\pm y) = (\cos x\cos y \mp \sin x\sin y) + \mathrm{i}(\sin x\cos y \pm \cos x\sin y)$

$\rightarrow\quad \cos(x\pm y) = \cos x\cos y \mp \sin x\sin y; \qquad \sin(x\pm y) = \sin x\cos y \pm \cos x\sin y.$

$\sin(2x) = \sin(x+x) = \sin x\cos x + \cos x\sin x = 2\sin x\cos x.$

$$\cos(2x) = \cos(x+x) = \cos x\cos x - \sin x\sin x = \cos^2 x - \sin^2 x$$
$$= \begin{cases}(1-\sin^2 x) - \sin^2 x = 1 - 2\sin^2 x\,; \\ \cos^2 x - (1-\cos^2 x) = 2\cos^2 x - 1.\end{cases}$$

$$\tan(x\pm y) = \frac{\sin(x\pm y)}{\cos(x\pm y)} = \frac{\sin x\cos y \pm \cos x\sin y}{\cos x\cos y \mp \sin x\sin y}$$ ← divide numerator and denominator by $\cos x\cos y$

$$= \left(\frac{\sin x\cancel{\cos y}}{\cos x\cancel{\cos y}} \pm \frac{\cancel{\cos x}\sin y}{\cancel{\cos x}\cos y}\right)\Big/\left(\frac{\cancel{\cos x}\cancel{\cos y}}{\cancel{\cos x}\cancel{\cos y}} \mp \frac{\sin x\sin y}{\cos x\cos y}\right) = \frac{\tan x \pm \tan y}{1 \mp \tan x\tan y}.$$