

**Solution to Problem 12)**

$$\begin{aligned}\exp(A) \exp(B) &= \left(\sum_{m=0}^{\infty} A^m/m!\right) \left(\sum_{n=0}^{\infty} B^n/n!\right) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A^m B^n / (m! n!) \\ &= \sum_{k=0}^{\infty} \sum_{n=0}^k A^{k-n} B^n / [n! (k-n)!] = \sum_{k=0}^{\infty} \left[ \sum_{n=0}^k \binom{k}{n} A^{k-n} B^n \right] / k!\end{aligned}$$

This step is valid if  $AB = BA$   $\rightarrow = \sum_{k=0}^{\infty} [(A + B)^k] / k! = \exp(A + B).$

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