Problem 1) a) The rectangular pulse has two discontinuities (or jumps), one at $x = -\frac{1}{2}\varepsilon$ and another at $x = \frac{1}{2}\varepsilon$. Upon differentiation with respect to x, the first jump produces a positive δ-function (magnitude $= 1/\varepsilon$), while the second gives rise to a negative δ-function. Consequently,

$$g'(x) = \mathrm{d}[\varepsilon^{-1}\mathrm{rect}(x/\varepsilon)]/\mathrm{d}x = \varepsilon^{-1}\delta(x + \tfrac{1}{2}\varepsilon) - \varepsilon^{-1}\delta(x - \tfrac{1}{2}\varepsilon).$$

b) $$\int_{-\infty}^{\infty} f(x)g'(x - x_0)\mathrm{d}x = \varepsilon^{-1}\int_{-\infty}^{\infty} f(x)\delta(x - x_0 + \tfrac{1}{2}\varepsilon)\mathrm{d}x - \varepsilon^{-1}\int_{-\infty}^{\infty} f(x)\delta(x - x_0 - \tfrac{1}{2}\varepsilon)\mathrm{d}x$$

sifting property of $\delta(x)$ → $$= \varepsilon^{-1}f(x_0 - \tfrac{1}{2}\varepsilon) - \varepsilon^{-1}f(x_0 + \tfrac{1}{2}\varepsilon)$$

definition of derivative ($\varepsilon \to 0$) → $$= -[f(x_0 + \tfrac{1}{2}\varepsilon) - f(x_0 - \tfrac{1}{2}\varepsilon)]/\varepsilon = -f'(x_0).$$