Problem 6) a) $\ln[\sin(\pi x)] = \ln(\pi x) + \sum_{n=1}^{\infty} \ln\left(1 - \frac{x^2}{n^2}\right).$

$$\frac{\mathrm{d}}{\mathrm{d}x}\ln[\sin(\pi x)] = \frac{\pi\cos(\pi x)}{\sin(\pi x)} = \frac{\pi}{\pi x} - \sum_{n=1}^{\infty} \frac{2x/n^2}{1-(x^2/n^2)} \quad \rightarrow \quad \cot(\pi x) = \frac{1}{\pi x} + \frac{2x}{\pi}\sum_{n=1}^{\infty}\left(\frac{1}{x^2-n^2}\right)$$

$$\rightarrow \quad \cot(\pi x) = \frac{1}{\pi x} + \frac{x}{\pi}\sum_{n=1}^{\infty}\left[\frac{1}{n(x-n)} - \frac{1}{n(x+n)}\right] = \frac{1}{\pi x} + \frac{x}{\pi}\sum_{\substack{n=-\infty \\ (n \neq 0)}}^{\infty}\left[\frac{1}{n(x-n)}\right].$$

b) Substituting $-\mathrm{i}x$ for x in the Euler expansion of $\sin(x)$ yields $\sinh(x) = x\prod_{n=1}^{\infty}\left(1 + \frac{x^2}{n^2\pi^2}\right)$. By the same token, the infinite-sum expansion of the hyperbolic cotangent function is the one given in the statement of the problem.