

Problem 2) a) $\frac{d}{dx} \sum_{n=0}^{\infty} x^n = \frac{d}{dx} (1-x)^{-1} \rightarrow \sum_{n=1}^{\infty} nx^{n-1} = (1-x)^{-2}.$

b)
$$\begin{aligned} (\sum_{n=0}^{\infty} x^n)^2 &= \sum_{m=0}^{\infty} x^m \sum_{n=0}^{\infty} x^n = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} x^{m+n} = \sum_{k=0}^{\infty} \sum_{n=0}^k x^k \\ &= \sum_{k=0}^{\infty} x^k \sum_{n=0}^k 1 = \sum_{k=0}^{\infty} (k+1)x^k = \sum_{n=1}^{\infty} nx^{n-1}. \end{aligned}$$
