Problem 1) $\sum_{n=1}^{N} n \cdot (n!) = 1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \cdots + N \cdot N!$ equals 1 for $N = 1$, which agrees with $(N+1)! - 1 = 2! - 1 = 1$. Also, for $N = 2$, the sum equals 5, which is the same as $3! - 1 = 5$. Assuming that the assertion is correct for N, we now show that it is also correct for $N + 1$. We will have

$$\sum_{n=1}^{N+1} n \cdot (n!) = [(N+1)! - 1] + (N+1) \cdot (N+1)! = (N+1)!\,(N+1+1) - 1 = (N+2)! - 1.$$

The proof is now complete. (Gradshteyn & Ryzhik 0.125)