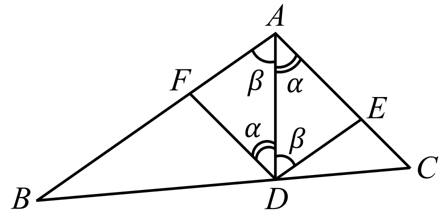
Problem 4) Drawing the lines DE and DF parallel to AB and AC, respectively, we note that the line AD cuts these pairs of parallel lines at equal angles, as shown. Since AD is the bisector of the angle A, we have $\alpha = \beta$. Consequently, $\overline{AE} = \overline{DE}$ and $\overline{DF} = \overline{AF}$. Considering that $AFDE$ is a parallelogram, we conclude that $\overline{DE} = \overline{DF}$.



Since the triangles ABC and FBD are similar, we have

$$\overline{BD} : \overline{BC} = \overline{DF} : \overline{AC}. \tag{1}$$

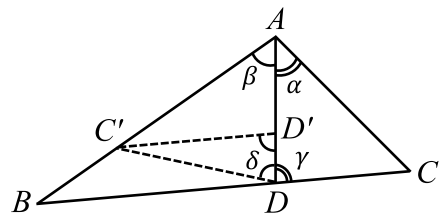
Similarly, since the triangles ABC and EDC are similar, one may write

$$\overline{CD} : \overline{BC} = \overline{DE} : \overline{AB}. \tag{2}$$

Dividing Eq.(1) by Eq.(2) and recalling that $\overline{DE} = \overline{DF}$ now yields the desired result, as follows:

$$\overline{BD} : \overline{CD} = \overline{AB} : \overline{AC}. \tag{3}$$

Digression: An alternative proof of the stated property of the triangle is outlined below.



Step 1: Pick the point C' on AB such that $\overline{AC'} = \overline{AC}$.

Step 2: Draw a straight line from D to C'. The equality of triangles ACD and $AC'D$ implies that $\overline{C'D} = \overline{CD}$ and $\widehat{ADC} = \widehat{ADC'}$ (these angles are identified as γ and δ).

Step 3: Draw the straight line $C'D'$ parallel to BD and observe that $\widehat{C'D'D} = \widehat{ADC} = \gamma$. Therefore, the triangle $DC'D'$ is isoceles, meaning that $\overline{C'D} = \overline{C'D'}$.

Step 4: The similar triangles $AC'D'$ and ABD now yield $\overline{AC'} : \overline{AB} = \overline{C'D'} : \overline{BD}$.

Considering that $\overline{AC'} = \overline{AC}$ and $\overline{C'D'} = \overline{C'D} = \overline{CD}$, we will have $\overline{AC} : \overline{AB} = \overline{CD} : \overline{BD}$, thus completing the proof.