Problem 4) Since ψ is the complement of φ, we have $\sin\psi = |\cos\varphi|$. (The absolute value of $\cos\varphi$ must be used here, in the event that $½\pi \le \varphi < \pi$, which would make $\cos\varphi$ negative.) It is seen that $h_1 = b\sin\theta$, and $h_2 = h_1\sin\psi = b\sin\theta\sin\psi$, and $h_3 = h_2\sin\psi = b\sin\theta\sin^2\psi$, and so on. So long as $\sin\psi < 1$ (i.e., $\psi \ne ½\pi$, or, equivalently, $0 < \varphi < \pi$), we may use the closed form of the resulting geometric series to arrive at

$$\textstyle\sum_{n=1}^{\infty} h_n = b\sin\theta\,(1 + \sin\psi + \sin^2\psi + \sin^3\psi + \cdots) = \frac{b\sin\theta}{1-\sin\psi} = \frac{b\sin\theta}{1-|\cos\varphi|}. \qquad (1)$$

Digression. The above formula can also be expressed in terms of the lengths a or c (instead of b) by invoking the well-known triangle identity $\frac{a}{\sin\theta} = \frac{b}{\sin\varphi} = \frac{c}{\sin(\theta+\varphi)}$, in which case we will have

$$\textstyle\sum_{n=1}^{\infty} h_n = \frac{b\sin\theta}{1-|\cos\varphi|} = \frac{a\sin\varphi}{1-|\cos\varphi|} = \frac{c\sin\theta\sin\varphi}{(1-|\cos\varphi|)\sin(\theta+\varphi)}. \qquad (2)$$

If we drag the vertex B to the left or the right along AB, the angle φ will increase or decrease without affecting b or θ. The first expression for $\sum h_n$ in Eq.(2) then reveals that $\sum h_n$ goes to infinity when φ approaches zero, and that it approaches $b\sin\theta$ when $\varphi \to ½\pi$. If φ rises above $½\pi$, the sum increases again and eventually goes to infinity as $\varphi \to \pi$.

If we drag the vertex A to the right or the left along the straight line AB (without changing a or φ), then, in accordance with the second expression in Eq.(2), $\sum h_n$ remains intact. And if we drag the vertex C upward along AD, then c remains constant while both θ and φ move toward $½\pi$, in which case the third expression in Eq.(2) indicates that $\sum h_n \to \infty$ as $\sin(\theta+\varphi) \to 0$. Similar arguments can be made if the vertex C moves one way or another along AC or along BC.