Problem 1) a) Taking into account all the multiple reflections within the gap, we will have

$$r = r_1 + t_1 r_2 t_1 + t_1 r_2 r_1 r_2 t_1 + t_1 r_2 (r_1 r_2)^2 t_1 + \cdots = r_1 + t_1 r_2 t_1 \sum_{n=0}^{\infty} (r_1 r_2)^n$$
$$= r_1 + \frac{t_1 r_2 t_1}{1 - r_1 r_2} = \frac{r_1 + (t_1^2 - r_1^2) r_2}{1 - r_1 r_2}. \quad (1)$$

b) In similar fashion, one writes the expression for the transmission coefficient through the bilayer slab, as follows:

$$t = t_1 t_2 + t_1 r_2 r_1 t_2 + t_1 (r_2 r_1)^2 t_2 + \cdots = t_1 t_2 \sum_{n=0}^{\infty} (r_2 r_1)^n = \frac{t_1 t_2}{1 - r_1 r_2}. \quad (2)$$

c) The probability amplitude t of the transmitted photon (also known as the transmission coefficient of the slab) as given by Eq.(2) would be precisely the same if the photon were to arrive from below, enter the bottom facet of the bilayer, then exit from the upper facet. This is because an exchange of t_1 with t_2, and, simultaneously, an exchange of r_1 with r_2, will not modify Eq.(2) at all. (This symmetric behavior of the transmission coefficient does *not*, in general, extend to the reflection coefficients from the opposite sides of the bilayer slab.)

Digression. In a more general case, we consider two slabs having Fresnel reflection and transmission coefficients (r_{1a}, t_{1a}) and (r_{2a}, t_{2a}) when illuminated from above and, similarly, (r_{1b}, t_{1b}) and (r_{2b}, t_{2b}) when illuminated from below. The light shines from above at normal incidence on the first slab, while the second slab sits immediately below the first. The Fresnel reflection and transmission coefficients of the bilayer slab are readily calculated, as follows:

$$r = r_{1a} + t_{1a} r_{2a} t_{1b} (1 + r_{2a} r_{1b} + r_{2a}^2 r_{1b}^2 + \cdots) = r_{1a} + \frac{r_{2a} t_{1a} t_{1b}}{1 - r_{2a} r_{1b}} = \frac{r_{1a} + (t_{1a} t_{1b} - r_{1a} r_{1b}) r_{2a}}{1 - r_{2a} r_{1b}}. \quad (3)$$

$$t = t_{1a} t_{2a} (1 + r_{2a} r_{1b} + r_{2a}^2 r_{1b}^2 + \cdots) = \frac{t_{1a} t_{2a}}{1 - r_{2a} r_{1b}}. \quad (4)$$

The transmissivity from above will be the same as that from below if $t_{1a} = t_{1b}$ and $t_{2a} = t_{2b}$. This will be true for individual slabs made up of homogeneous materials. Repeating the argument and adding more and more homogeneous slabs proves that, for any multilayer stack, the transmission coefficients from above and below will be the same. In general, the reflectivities from the opposite sides will *not* be the same if one or more of the layers are partially absorptive.