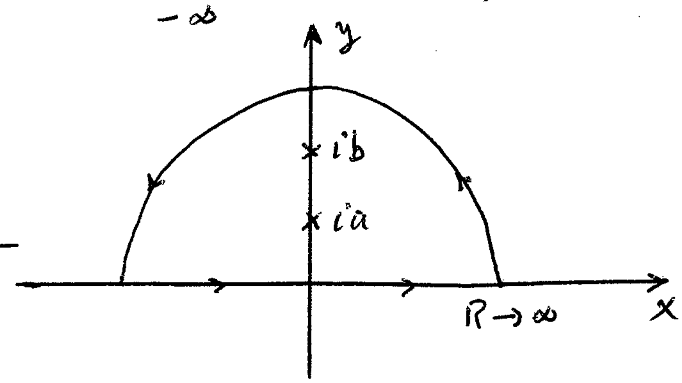
Problem 6) $$\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)} = \frac{1}{2}\int_{-\infty}^{\infty} \frac{dx}{(x^2+a^2)(x^2+b^2)}$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} \frac{dx}{(x+ia)(x-ia)(x+ib)(x-ib)}$$



Residue at $z = ia$: $$\frac{1}{2ia(ia+ib)(ia-ib)} = \frac{i}{2a(a^2-b^2)}$$

Residue at $z = ib$: $$\frac{1}{(ib+ia)(ib-ia)2ib} = \frac{i}{2b(b^2-a^2)}$$

$$\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)} = \frac{1}{2}\, 2\pi i \left[\frac{i}{2a(a^2-b^2)} + \frac{i}{2b(b^2-a^2)} \right]$$

$$= -\pi \frac{b-a}{2ab(a^2-b^2)} = \frac{\pi}{2ab(a+b)}$$

We also need to show that the integral on the semi-circle goes to zero as $R \to \infty$.

$$\left| \oint_{\text{Semi-Circle}} \frac{dz}{(z^2+a^2)(z^2+b^2)} \right| = \left| \int_{\theta=0}^{\pi} \frac{iRe^{i\theta}d\theta}{(R^2e^{2i\theta}+a^2)(R^2e^{2i\theta}+b^2)} \right|$$

$$\leq \int_{\theta=0}^{\pi} \frac{|iRe^{i\theta}d\theta|}{|R^2e^{2i\theta}+a^2|\,|R^2e^{2i\theta}+b^2|} = R\int_0^{\pi} \frac{d\theta}{|R^2+a^2e^{-2i\theta}|\,|R^2+b^2e^{-2i\theta}|}$$

$$\leq R\int_0^{\pi} \frac{d\theta}{(R^2-a^2)(R^2-b^2)} = \frac{R\pi}{(R^2-a^2)(R^2-b^2)} \xrightarrow[R\to\infty]{} 0.$$