Problem 5) We apply the method of stationary-phase with $f(x)=\exp(-x^2)$ and $g(x)=\sqrt{1-x^2}$ to this integral. The stationary point is found by setting $g'(x)=-x(1-x^2)^{-1/2}=0$, from which the sole stationary point is found to be $x_o=0$. This is an acceptable stationary point as it resides within the $(-1,1)$ integration range. At the stationary point, the value of $f(x)$ is 1, that of $g(x)$ is also 1, and the value of the 2nd derivative $g''(x)=-(1-x^2)^{-3/2}$ at $x=x_o$ is found to be -1. We have shown in the class that the approximate value of the integral is

$$I \approx \sqrt{\frac{2\pi}{|\eta g''(x_o)|}}\, f(x_o)\exp[i\eta g(x_o)]\exp(\pm i\pi/4); \qquad \pm \text{ depending on whether } \eta g''(x_o) \gtrless 0.$$

Substitution for the various parameters in the above expression thus yields

$$I \approx \sqrt{2\pi/\eta}\,\exp[i(\eta-\pi/4)].$$

It is instructive to numerically evaluate the integral for large values of η to see how close the approximate value comes to the actual value of the integral. A Matlab® program for such a calculation is listed below.

```
%
% Evaluating the Stationary-Phase Approximation to an Integral
%
clear all
eta=25;
nmax=1000;
deltax=2/nmax;
%
I=0;
for n=1:nmax
    x=-1+(n-1)*deltax;
    x2=x*x;
    I=I+exp(-x2+i*eta*sqrt(1-x2));
end
Integral=I*deltax
%
Approximation=sqrt(2*pi/eta)*exp(i*(eta-pi/4))
%
%End of Program
```