Problem 4)

$$f(p_1, p_2, \cdots, p_N) = -\sum_{n=1}^{N} p_n \ln(p_n) + \lambda \sum_{n=1}^{N} p_n.$$

$$\frac{\partial f}{\partial p_n} = -\ln(p_n) - 1 + \lambda = 0 \quad \rightarrow \quad p_n = e^{\lambda-1} \qquad (n = 1, 2, 3, \cdots, N).$$

All p_n are therefore equal to $e^{\lambda-1}$, which makes them equal to each other. Considering that $\sum_{n=1}^{N} p_n = 1$, we conclude that $p_1 = p_2 = \cdots = p_N = 1/N$. The Shannon entropy $H(p_1, p_2, \ldots, p_N)$ is thus maximized when all the various outcomes of the experiment are equally likely. The maximum entropy is given by $-\sum_{n=1}^{N} p_n \ln(p_n) = -\sum_{n=1}^{N} N^{-1} \ln(N^{-1}) = \ln N$.