Problem 4) For a given value of n, we are asked to maximize the product $x_1x_2\cdots x_n$ subject to the constraint $x_1+x_2+\cdots+x_n=L$. In accordance with the method of Lagrange multipliers, we form the function $f(x_1, x_2,\ldots, x_n)=x_1x_2\cdots x_n+\lambda(x_1+x_2+\cdots+x_n)$, then set its derivatives with respect to $x_1, x_2,\ldots, x_n$ equal to zero, as follows:

$$\partial f/\partial x_1 = x_2x_3\cdots x_n + \lambda = 0 \quad\rightarrow\quad x_2x_3\cdots x_n = -\lambda;$$

$$\partial f/\partial x_2 = x_1x_3\cdots x_n + \lambda = 0 \quad\rightarrow\quad x_1x_3\cdots x_n = -\lambda;$$

$$\vdots$$

$$\partial f/\partial x_n = x_1x_2\cdots x_{n-1} + \lambda = 0 \quad\rightarrow\quad x_1x_2\cdots x_{n-1} = -\lambda.$$

Dividing the first of the above equations by the second yields $x_2/x_1=1$, revealing that x_1 and x_2 must be equal. Similarly, dividing the first equation by the third shows that $x_1=x_3$, and so on. The solution of the above equations is thus given by $x_1 = x_2 = \cdots = x_n = \sqrt[n]{-\lambda}$. In the next step, we substitute the above solution into the constraint equation $x_1+x_2+\cdots+x_n=L$ to find the value of λ. We will have $n\sqrt[n]{-\lambda} = L$, which yields $\lambda=-(L/n)^n$. The optimum solution for the lengths of the various segments is thus found to be $x_1 = x_2 = \cdots = x_n = \sqrt[n]{-\lambda} = L/n$.