Problem 4) $f(z) = f(x+iy) = (x-y)^2 + 2i(x+y)$.

$u(x,y) = (x-y)^2, \quad v(x,y) = 2(x+y)$

Cauchy-Riemann Conditions:

$$\frac{\partial u}{\partial x} = 2(x-y); \quad \frac{\partial u}{\partial y} = -2(x-y); \quad \frac{\partial v}{\partial x} = 2; \quad \frac{\partial v}{\partial y} = 2.$$

$$\begin{cases} \dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y} \Rightarrow 2(x-y) = 2 \Rightarrow x-y=1. \\ \dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x} \Rightarrow -2(x-y) = -2 \Rightarrow x-y=1. \end{cases}$$

Clearly, the straight-line $x-y=1$ is the only place within the complex-plane z where the function $f(z)$ is differentiable.

At no point on the straight-line $x-y=1$ can we find a small neighborhood (i.e., a small circle centered at that point) where the function is differentiable everywhere within the neighborhood. Therefore $f(z)$ is analytic at <u>no</u> point.