Problem 4)

a) $f(x) = \text{Tri}(x)$.

b) $$F(s) = \int_{-\infty}^{\infty} f(x)\exp(-\text{i}2\pi sx)\,dx = ½\int_0^2\int_{-\infty}^{\infty}\text{Rect}(x/\beta)\exp(-\text{i}2\pi sx)\,dxd\beta$$

Using scaling property and the fact that $\mathcal{F}\{\text{Rect}(x)\} = \text{sinc}(s)$. →

$$= ½\int_0^2 \beta\,\text{sinc}(\beta s)\,d\beta = ½\int_0^2 \beta\frac{\sin(\pi\beta s)}{\pi\beta s}d\beta = \frac{1}{2\pi s}\int_0^2 \sin(\pi\beta s)\,d\beta$$

$$= -\frac{\cos(\pi\beta s)}{2(\pi s)^2}\bigg|_{\beta=0}^{2} = \frac{1-\cos(2\pi s)}{2(\pi s)^2} = \frac{2\sin^2(\pi s)}{2(\pi s)^2} = \text{sinc}^2(s).$$

It is thus confirmed that the Fourier transform of $f(x)$ is the same as that of $\text{Tri}(x)$.