Problem 4) Since $h(x)$ is defined over the interval $[0, L]$, and since the desired Fourier cosine series is an even function of x whose period is $2L$, we must first extend $h(x)$ over the interval $[-L, +L]$ in such a way as to create an even function $\underset{\sim}{h}(x)$. This is done by defining $\underset{\sim}{h}(x) = h(|x|)$. We then proceed to determine the ordinary Fourier series of the function $\underset{\sim}{h}(x)$, as follows:

$$\underset{\sim}{H}(s) = \mathscr{F}\{\underset{\sim}{h}(x)\} = \int_{-L}^{L} \underset{\sim}{h}(x)\exp(-\mathrm{i}2\pi sx)\mathrm{d}x = \int_{-L}^{0} h(-x)\exp(-\mathrm{i}2\pi sx)\mathrm{d}x + \int_{0}^{L} h(x)\exp(-\mathrm{i}2\pi sx)\mathrm{d}x$$

$$= \int_{0}^{L} h(x)\exp(+\mathrm{i}2\pi sx)\mathrm{d}x + \int_{0}^{L} h(x)\exp(-\mathrm{i}2\pi sx)\mathrm{d}x = 2\int_{0}^{L} h(x)\cos(2\pi sx)\mathrm{d}x.$$

When $\underset{\sim}{h}(x)$ is turned into a periodic function with period $2L$ (using convolution with a comb function), its Fourier series coefficients become

$$\underset{\sim}{c}_n = \frac{1}{2L}\underset{\sim}{H}\left(\frac{n}{2L}\right) = \frac{1}{L}\int_{0}^{L} h(x)\cos(n\pi x/L)\mathrm{d}x.$$

Taking advantage of the fact that $\underset{\sim}{c}_{-n} = \underset{\sim}{c}_n$, we may now write the Fourier series of $\underset{\sim}{h}(x)$ over the interval $[-L, +L]$ as follows:

$$\underset{\sim}{h}(x) = \sum_{n=-\infty}^{\infty} \underset{\sim}{c}_n \exp[\mathrm{i}2\pi nx/(2L)] = \underset{\sim}{c}_0 + \sum_{n=1}^{\infty} \underset{\sim}{c}_n [\exp(\mathrm{i}\pi nx/L) + \exp(-\mathrm{i}\pi nx/L)]$$

$$= \underset{\sim}{c}_0 + \sum_{n=1}^{\infty} 2\underset{\sim}{c}_n \cos(n\pi x/L).$$

Comparison with the Fourier cosine series expansion of $h(x)$ over the interval $[0, L]$ now yields

$$c_0 = \underset{\sim}{c}_0 = \frac{1}{L}\int_{0}^{L} h(x)\mathrm{d}x,$$

$$c_n = 2\underset{\sim}{c}_n = \frac{2}{L}\int_{0}^{L} h(x)\cos(n\pi x/L)\mathrm{d}x.$$