Problem 3)

a) $f(x) = [\alpha^{-1}\text{rect}(x/\alpha) * p^{-1}\text{comb}(x/p)] \times \text{rect}[x/(2N+1)p]$.

b) Area under the square of the function: $\int_{-\infty}^{\infty} f^2(x)\mathrm{d}x = (2N+1)/\alpha$.

c) $F(s) = [\text{sinc}(\alpha s)\text{comb}(ps)] * (2N+1)p\,\text{sinc}[(2N+1)ps]$ ← Use convolution, multiplication, and scaling theorems.

$= (2N+1)\sum_{n=-\infty}^{\infty}\text{sinc}(n\alpha/p)\delta(s-n/p) * \text{sinc}[(2N+1)ps]$

$= (2N+1)\sum_{n=-\infty}^{\infty}\text{sinc}(n\alpha/p)\text{sinc}[(2N+1)(ps-n)]$. ← Convolution with $\delta(\cdot)$ shifts the $\text{sinc}(\cdot)$ to the location of $\delta(\cdot)$.

d) $\int_{-\infty}^{\infty} F^2(s)\mathrm{d}s \cong (2N+1)^2\sum_{n=-\infty}^{\infty}\text{sinc}^2(n\alpha/p)\int_{-\infty}^{\infty}\text{sinc}^2[(2N+1)(ps-n)]\mathrm{d}s$

↑ Overlap among $\text{sinc}^2(\cdot)$ functions for different values of n is being ignored.

$= \frac{(2N+1)^2}{(2N+1)p}\sum_{n=-\infty}^{\infty}\text{sinc}^2(n\alpha/p)$

$= [(2N+1)/\alpha]\sum_{n=-\infty}^{\infty}(\alpha/p)\text{sinc}^2(n\alpha/p)$ ← Treat this sum is an approximation to the area under $\text{sinc}^2(s)$, sampled at regular α/p intervals.

$\cong [(2N+1)/\alpha]\int_{-\infty}^{\infty}\text{sinc}^2(s)\mathrm{d}s = (2N+1)/\alpha$.

e) The areas under the square of the function and the square of its Fourier transform are identical, even though both go to infinity when $\alpha \to 0$ and $N \to \infty$. Needless to say, Parseval's theorem is valid for any values of N and α, but the above argument does not demonstrate its universal validity if the adjacent sinc functions happen to have significant overlap. It is possible, however, to prove, using the same parameters as we have chosen here, that the overlap integrals between a $\text{sinc}^2(\cdot)$ function and the tails of all the other $\text{sinc}^2(\cdot)$ functions are exactly zero, and that the aforementioned approximation to the area under $\text{sinc}^2(s)$ is, in fact, exact, provided that $\alpha \le p$.