

Problem 2)

Binomial expansion: $(1+x)^n = \sum_{k=0}^n \binom{n}{k} x^k$.

Integrating the above equation over x from 0 to 1 yields

$$\begin{aligned} \int_0^1 (1+x)^n dx &= \sum_{k=0}^n \binom{n}{k} \int_0^1 x^k dx \quad \rightarrow \quad \left. \frac{(1+x)^{n+1}}{n+1} \right|_{x=0}^1 = \sum_{k=0}^n \binom{n}{k} \left. \frac{x^{k+1}}{k+1} \right|_{x=0}^1 \\ &\rightarrow \frac{2^{n+1} - 1}{n+1} = \sum_{k=0}^n \frac{1}{k+1} \binom{n}{k}. \end{aligned}$$
