

Opht 503

Solutions

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Problem 2) $f(x) = \sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$; $|x| < 1$ ← Geometric Series

Taking derivatives from both sides of the above equation yields:

$$f'(x) = \sum_{n=0}^{\infty} n x^{n-1} = \frac{1}{(1-x)^2} \Rightarrow \sum_{n=1}^{\infty} n x^{n-1} = \frac{1}{(1-x)^2} \leftarrow \begin{array}{l} \text{because the} \\ \text{first term at} \\ n=0 \text{ is zero.} \end{array}$$

Multiplying both sides by x then yields:

$$\sum_{n=1}^{\infty} n x^n = \frac{x}{(1-x)^2}, \quad |x| < 1$$