

Problem 1) $f(x) = x \text{Rect}(x/2)$

$$\mathcal{F}\{\text{Rect}(x)\} = \text{sinc}(s) \xrightarrow{\substack{\text{scaling} \\ \text{theorem}}} \mathcal{F}\{\text{Rect}(x/2)\} = 2 \text{sinc}(2s) = \frac{\sin(2\pi s)}{\pi s}$$

$$G(s) = \int_{-\infty}^{\infty} g(x) e^{-i2\pi s x} dx \Rightarrow \frac{dG(s)}{ds} = \int_{-\infty}^{\infty} -i2\pi x g(x) e^{-i2\pi s x} dx \Rightarrow$$

$$\mathcal{F}\{x g(x)\} = \frac{i}{2\pi} \frac{dG(s)}{ds} \quad \leftarrow \text{differentiation theorem}$$

$$\text{Therefore, } \mathcal{F}\{x \text{Rect}(x/2)\} = \frac{i}{2\pi} \frac{d}{ds} \left[\frac{\sin(2\pi s)}{\pi s} \right] = \frac{i}{2\pi^2} \frac{2\pi s \cos(2\pi s) - \sin(2\pi s)}{s^2}$$

$$\Rightarrow \mathcal{F}\{x \text{Rect}(x/2)\} = \frac{i \cos(2\pi s)}{\pi s} - \frac{i \sin(2\pi s)}{2\pi^2 s^2}$$

$$F(s) = \mathcal{F}\{f(x)\} = \int_{-\infty}^{\infty} f(x) e^{-i2\pi s x} dx = \int_{-1}^{+1} x e^{-i2\pi s x} dx$$

$$\xrightarrow{\substack{\text{integration} \\ \text{by parts}}} = \frac{x}{-i2\pi s} e^{-i2\pi s x} \Big|_{x=-1}^{+1} + \frac{1}{i2\pi s} \int_{-1}^{+1} e^{-i2\pi s x} dx$$

$$= \frac{i}{2\pi s} (e^{-i2\pi s} + e^{+i2\pi s}) - \frac{1}{(i2\pi s)^2} e^{-i2\pi s x} \Big|_{x=-1}^{+1}$$

$$= \frac{i \cos(2\pi s)}{\pi s} + \frac{1}{4\pi^2 s^2} (e^{-i2\pi s} - e^{+i2\pi s})$$

$$= \frac{i \cos(2\pi s)}{\pi s} - \frac{i \sin(2\pi s)}{2\pi^2 s^2}$$