

Problem 1)

a)
$$\exp(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots$$

b)
$$\begin{aligned} \exp(ix) &= 1 + ix + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \dots + \frac{(ix)^n}{n!} + \dots \\ &= \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots\right) + i \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots\right) \end{aligned}$$

Considering that $\exp(ix) = \cos x + i \sin x$, it is clear that, in the preceding expression, the first sum (the real part) is equal to $\cos x$, while the second sum (the imaginary part) is equal to $\sin x$. Setting $x = 1$ radian, we will have

c)
$$\cos(1) = 0.54030231 = 1 - \frac{1}{2!} + \frac{1}{4!} - \frac{1}{6!} + \frac{1}{8!} - \dots$$

d)
$$\sin(1) = 0.84147098 = 1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \frac{1}{9!} - \dots$$
