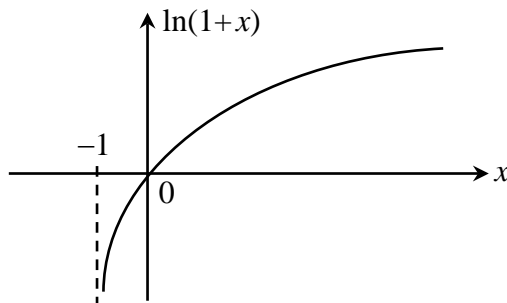


## Problem 1)

a)



$$\left. \frac{d \ln(1+x)}{dx} \right|_{x=0} = (1+x)^{-1} \Big|_{x=0} = 1$$

$$\left. \frac{d^2 \ln(1+x)}{dx^2} \right|_{x=0} = -(1+x)^{-2} \Big|_{x=0} = -1$$

$$\left. \frac{d^3 \ln(1+x)}{dx^3} \right|_{x=0} = 2(1+x)^{-3} \Big|_{x=0} = 2!$$

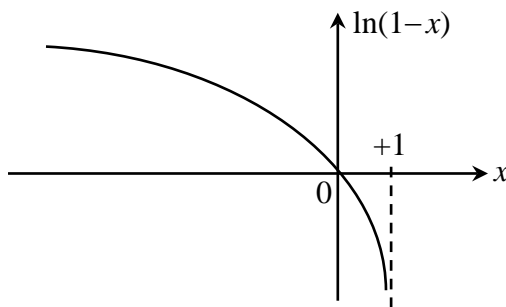
$$\left. \frac{d^4 \ln(1+x)}{dx^4} \right|_{x=0} = -3!(1+x)^{-4} \Big|_{x=0} = -3!$$

$$\vdots$$

Therefore,

$$\begin{aligned} \ln(1+x) &= \ln(1) + x - \frac{1 \times x^2}{2!} + \frac{2! \times x^3}{3!} - \frac{3! \times x^4}{4!} + \dots \\ &= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \end{aligned}$$

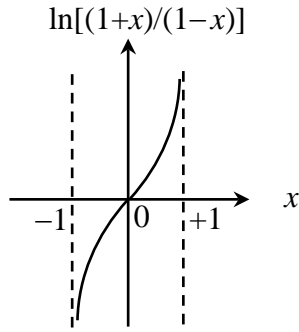
b)



In this case the solution is obtained by changing the sign of  $x$  in the Taylor-series expansion of  $\ln(1+x)$  obtained in part (a), that is,

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \dots$$

$$\begin{aligned} \text{c) } \ln\left(\frac{1+x}{1-x}\right) &= \ln(1+x) - \ln(1-x) = \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots\right) - \left(-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \dots\right) \\ &= 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \dots\right). \end{aligned}$$



d) Setting  $x = \frac{1}{2}$  in the above Taylor-series expansion of  $\ln\left(\frac{1+x}{1-x}\right)$ , we find

$$\ln\left(\frac{1+x}{1-x}\right)\bigg|_{x=\frac{1}{2}} = \ln\left(\frac{3/2}{1/2}\right) = \ln 3 = 2\left(\frac{1}{2} + \frac{1}{3 \times 2^3} + \frac{1}{5 \times 2^5} + \frac{1}{7 \times 2^7} + \dots\right).$$

e) Setting  $x = \frac{1}{3}$  in the Taylor-series expansion of  $\ln\left(\frac{1+x}{1-x}\right)$ , we find

$$\ln\left(\frac{1+x}{1-x}\right)\bigg|_{x=\frac{1}{3}} = \ln\left(\frac{4/3}{2/3}\right) = \ln 2 = 2\left(\frac{1}{3} + \frac{1}{3 \times 3^3} + \frac{1}{5 \times 3^5} + \frac{1}{7 \times 3^7} + \dots\right).$$


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