

**Problem 1)** The function  $\alpha f(\alpha x) = \alpha/\cosh(\pi\alpha x)$ , where  $\alpha > 0$  is an arbitrary real-valued parameter, is a scaled version of  $f(x)$ . The scaling theorem of Fourier transform shows that this function is transformed to  $F(s/\alpha) = 1/\cosh(\pi s/\alpha)$ . The area under  $\alpha f(\alpha x)$  is given by its Fourier transform  $F(s/\alpha)$  at  $s = 0$ ; that is  $\int_{-\infty}^{\infty} \alpha f(\alpha x) dx = 1/\cosh(0) = 1$ .

As  $\alpha \rightarrow \infty$ , the height of  $\alpha f(\alpha x)$ , namely,  $\alpha f(0) = \alpha$ , goes to infinity, while its width shrinks toward zero, but the function remains symmetric, and its area remains constant at 1. This means that  $\alpha f(\alpha x)$  approaches  $\delta(x)$  in the limit when  $\alpha \rightarrow \infty$ . In the same limit,  $F(s/\alpha)$  retains its height of 1, while its width expands to infinity; in other words,  $F(s/\alpha) \rightarrow 1$  when  $\alpha \rightarrow \infty$ . We conclude that the Fourier transform of  $g(x) = \delta(x)$  is  $G(s) = 1$ .

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