

Problem 1)

$\int_0^{\infty} \frac{dx}{(x^2+a^2)^2}$  ← Use an infinitely large semi-circular contour in the upper half-plane.

Poles:  $z^2+a^2=0 \Rightarrow z=\pm ia$

The pole inside the contour is  $z=ia$ , and it is a 2<sup>nd</sup>-order pole.

Residue at  $z=ia$ :  $\left( \frac{1}{(z+ia)^2} \right)' \Big|_{z=ia} = -2(z+ia)^{-3} \Big|_{z=ia} = -2(2ia)^{-3} = \frac{1}{i4a^3}$

$$\int_0^{\infty} \frac{dx}{(x^2+a^2)^2} = \frac{1}{2} \int_{-\infty}^{\infty} \frac{dx}{(x^2+a^2)^2} = \frac{1}{2} (2\pi i) \left( \frac{1}{i4a^3} \right) = \frac{\pi}{4a^3} \quad \checkmark$$