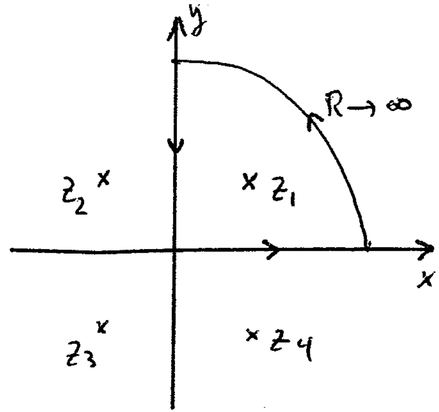
Problem 1) $\int_0^\infty \frac{dx}{x^4+4a^4}$



Poles: $z^4+4a^4=0 \Rightarrow z^4=-4a^4 \Rightarrow z^2=\pm i2a^2$

$$\Rightarrow z^2=2a^2e^{\pm i\pi/2} \Rightarrow z=\pm\sqrt{2}\,a\,e^{\pm i\pi/4}$$

The only pole inside the contour is $z_1=\sqrt{2}\,a\,e^{i\pi/4}$

$$\text{Residue at } z_1 = \frac{1}{(z_1-z_2)(z_1-z_3)(z_1-z_4)} = \frac{1}{2a\cdot 2\sqrt{2}\,a\,e^{i\pi/4}\cdot 2ai}$$

$$= \frac{1}{i\,8\sqrt{2}\,a^3e^{i\pi/4}}.$$

Loop integral when $R\to\infty$: $\int_0^\infty \frac{dx}{x^4+4a^4} - \int_0^\infty \frac{i\,dy}{(iy)^4+4a^4} = (1-i)\int_0^\infty \frac{dx}{x^4+4a^4}$

$$= 2\pi i\left(\frac{1}{i\,8\sqrt{2}\,a^3e^{i\pi/4}}\right) \Rightarrow \int_0^\infty \frac{dx}{x^4+4a^4} = \frac{\pi}{4\sqrt{2}\,a^3e^{i\pi/4}(1-i)} = \frac{\pi}{8a^3}.$$