Problem 1) Poles: $z^4+1=0 \Rightarrow z^4=-1=e^{i(2n+1)\pi} \Rightarrow z=e^{i(2n+1)\pi/4}$.

There are four differet poles as follows: $z_1=e^{i\pi/4}$, $z_2=e^{i3\pi/4}$, $z_3=e^{i5\pi/4}$

and $z_4=e^{i7\pi/4}$. Only z_1 fall within the chosen contour.

$$\text{Residue at } z_1 = \frac{z_1}{(z_1-z_2)(z_1-z_3)(z_1-z_4)} = \frac{e^{i\pi/4}}{(e^{i\pi/4}-e^{i3\pi/4})(e^{i\pi/4}-e^{i5\pi/4})(e^{i\pi/4}-e^{i7\pi/4})}$$

$$= \frac{e^{i\pi/4}}{\sqrt{2}\cdot 2e^{i\pi/4}\cdot\sqrt{2}\,i} = \frac{1}{4i}.$$

We also note that $\int \frac{z\,dz}{z^4+1}$ approaches zero as $R\to\infty$ on the circular arc.

Therefore, $$\int_0^\infty \frac{x\,dx}{x^4+1} - \int_0^\infty \frac{iy}{(iy)^4+1}\,i\,dy = 2\pi i\left(\frac{1}{4i}\right) \Rightarrow$$

$$\int_0^\infty \frac{x\,dx}{x^4+1} + \int_0^\infty \frac{y\,dy}{y^4+1} = \frac{\pi}{2} \Rightarrow \int_0^\infty \frac{x\,dx}{x^4+1} = \frac{\pi}{4}.$$