Problem 4) Let the area of the rectangle be denoted by $f(x,y) = xy$, while its perimeter is given by $g(x,y) = 2(x+y) = P$. Setting the partial derivatives with respect to x and y of the composite function $f + \lambda g$ equal to zero, we find

$$\partial_x(f + \lambda g) = y + 2\lambda = 0 \quad \rightarrow \quad y = -2\lambda,$$

$$\partial_y(f + \lambda g) = x + 2\lambda = 0 \quad \rightarrow \quad x = -2\lambda.$$

Consequently, $g(x,y) = 2(-2\lambda - 2\lambda) = -8\lambda = P$, yielding $\lambda = -P/8$. From here we find $x = y = -2\lambda = P/4$. The rectangle having the maximum area is thus a square, with each side being equal to $P/4$.