Solution to Problem 4) The composite function is $A(r_1, r_2, \cdots, r_N) + \lambda P(r_1, r_2, \cdots, r_N)$. Setting the partial derivative of the composite function with respect to r_n equal to zero, we will have

$$\partial_{r_n} A + \lambda \partial_{r_n} P = r_n \Delta\theta + \lambda \Delta\theta = 0 \quad \rightarrow \quad r_n = -\lambda.$$

Next, we enforce the constraint $P = P_0$. We find $P = \sum r_n \Delta\theta = -\lambda \sum \Delta\theta = -2\pi\lambda = P_0$, which yields $\lambda = -P_0/2\pi$. We will then have $r_1 = r_2 = \cdots = r_N = -\lambda = P_0/2\pi$. The curve which encloses the maximum area A for a given perimeter $P = P_0$ is a circle of radius $r = P_0/2\pi$.